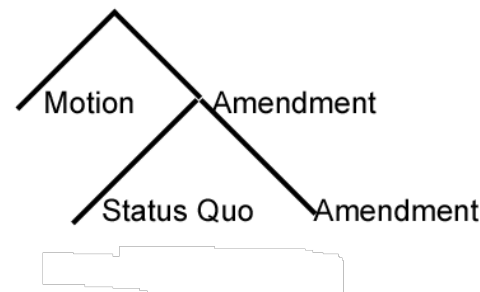


# **THE MEDIAN VOTER THEOREM (ONE DIMENSION)**



# Single Dimensional Spatial Model

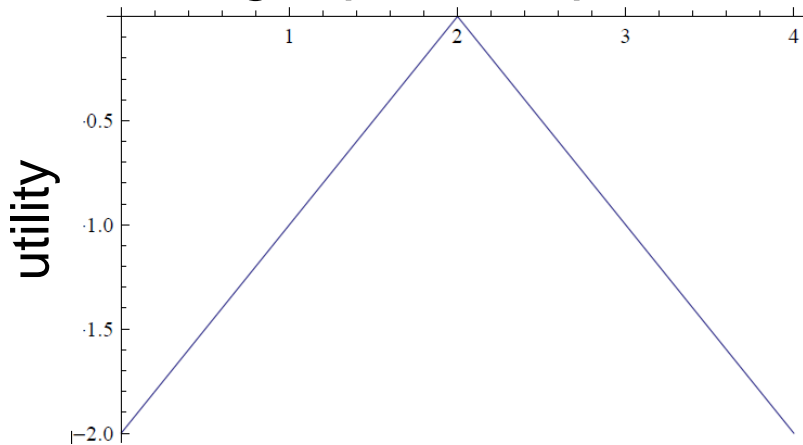
- Alternatives are the set of points on a line
  - Various ideologies on a spectrum
  - Spending on different programs
  - etc.
- Single-peaked preferences
  - Preferences are **satiabile**
  - Each agent has an **ideal point** (most-preferred alternative)
- Symmetric preferences
  - Individuals prefer alternatives closer to their ideal point more than those farther away.
- Pairwise majority rule
  - Alternatives will be considered two at a time using majority rule.



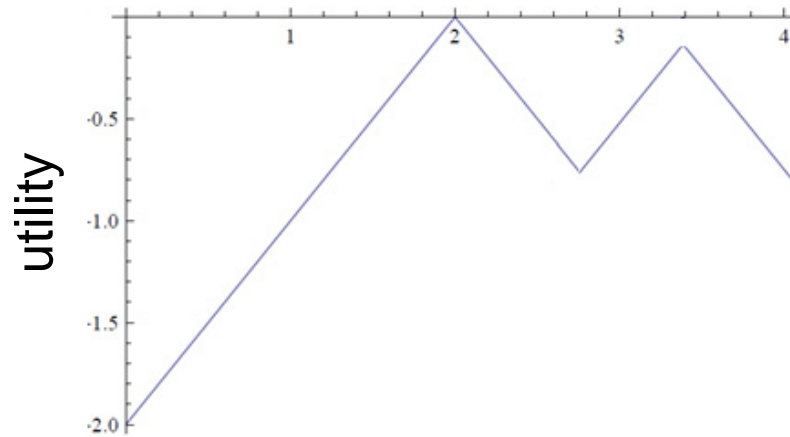
# Single Peaked Preferences

With single peaked preferences, utility is a decreasing function of the distance between the alternative and the ideal point.

- Single peaked preferences



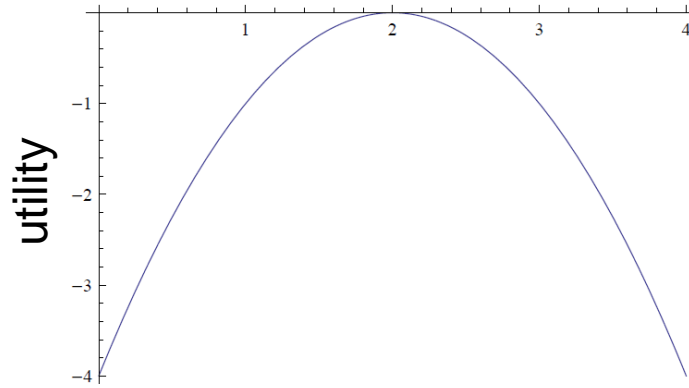
- Non-single peaked preferences



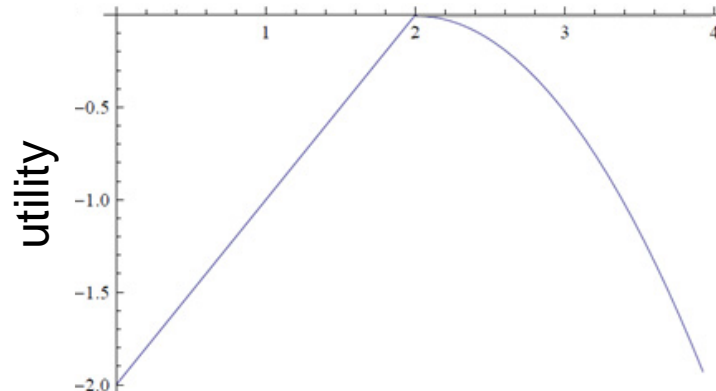
# Symmetric Preferences

With symmetric preferences, individuals prefer alternatives closer to their ideal point more than alternatives farther away.

- Symmetric preferences



- Asymmetric preferences

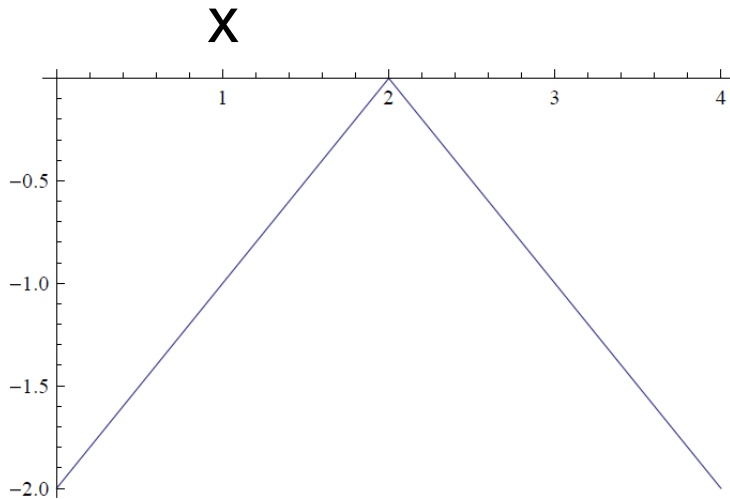


# Preferred-to set

The **preferred-to set** of  $x$  is the set of points  $y$  such that  $yPx$

$$P(x) = \{y \in X \mid U(y) > U(x)\}$$

What does the voter  
at 2 prefer to  $x$ ?



# Example

Suppose policy is one-dimensional and that a legislator has single-peaked and symmetric preferences with an ideal point at 2. If the status quo policy is located at 7, what is the set of policies that the legislator prefers to the status quo?

Suppose policy is one-dimensional and that a legislator has single-peaked and symmetric preferences with an ideal point at 9. If the status quo policy is located at 7, what is the set of policies that the legislator prefers to the status quo?

# Equilibrium Concept

- **Core:** Alternative  $x$  is an element of the core of an  $f$ -voting rule game if there does not exist another alternative  $y$  that  $f$  individuals prefer to  $x$ .
  - Ex:  $x$  is an element of the majority rule core if there does not exist another alternative  $y$  that a majority of individuals prefer to  $x$ .
  - The core is an equilibrium concept for spatial voting games.



# Median Voter Theorem

Given:

- 1)  $n > 2$  (and  $n$  is odd),
- 2) pairwise majority rule voting,
- 3) alternatives are on a single dimension,
- 4) preferences are single peaked.

**MVT.** The core is the median voter's ideal point under pairwise majority rule (i.e. the median voter's position is in equilibrium).

Remarks

- Also works for  $n$  even, you just have to understand what we mean by “median.”
- Median voter = individual such that half of the other ideal points are opposite sides of the median's ideal point.

# Proof of The Median Voter Theorem ( $n$ odd)

## Notation

$t_m$  = median's ideal point

$q$  = the status quo.

$L = (n-1)/2$  number of ideal points to the left of  $t_m$

$R = (n-1)/2$  number of ideal points to the right of  $t_m$

Assume  $q = t_m$ . First show that  $q$  is in the core.

Consider an arbitrary  $x$  such that  $x < t_m$ . Note that  $R \cup \{t_m\}$  individuals prefer  $q$  to  $x$ ; thus, a majority do not not prefer  $x$  to  $q$ .

Consider an arbitrary  $y$  such that  $y > t_m$ . Note that  $L \cup \{t_m\}$  individuals prefer  $q$  to  $x$ ; thus, a majority cannot not prefer  $y$  to  $q$ .

The proof follows by showing that any  $z \neq t_m$  is *not* in the core, which follows because  $q$  will attain either  $R \cup \{t_m\}$  votes or  $L \cup \{t_m\}$  votes, and defeat  $z$ .

# Proof of The Median Voter Theorem ( $n$ even)

Prove the MVT for  $n$  even.

- Order the voters ideal points from smallest to largest and note that the median pivots are in position  $M_1 = n/2$  and  $M_2 = (n+2)/2$ . The total number of voters to the right (larger) than  $M_2$  are  $n - (n+2)/2 = (n - n/2) - 1 = n/2 - 1$ . This means that there is less than a majority of the members to the right of  $M_2$  (larger than  $M_2$ ). Hence, any alternative to the right of  $M_2$  (larger) cannot receive majority of votes in favor of it.
- Similar reasoning shows that there is not a majority of individuals to the left (smaller) than  $M_1$ .
- Hence, for  $n$  even, the core is  $[M_1, M_2]$ .
  - EX: on board.

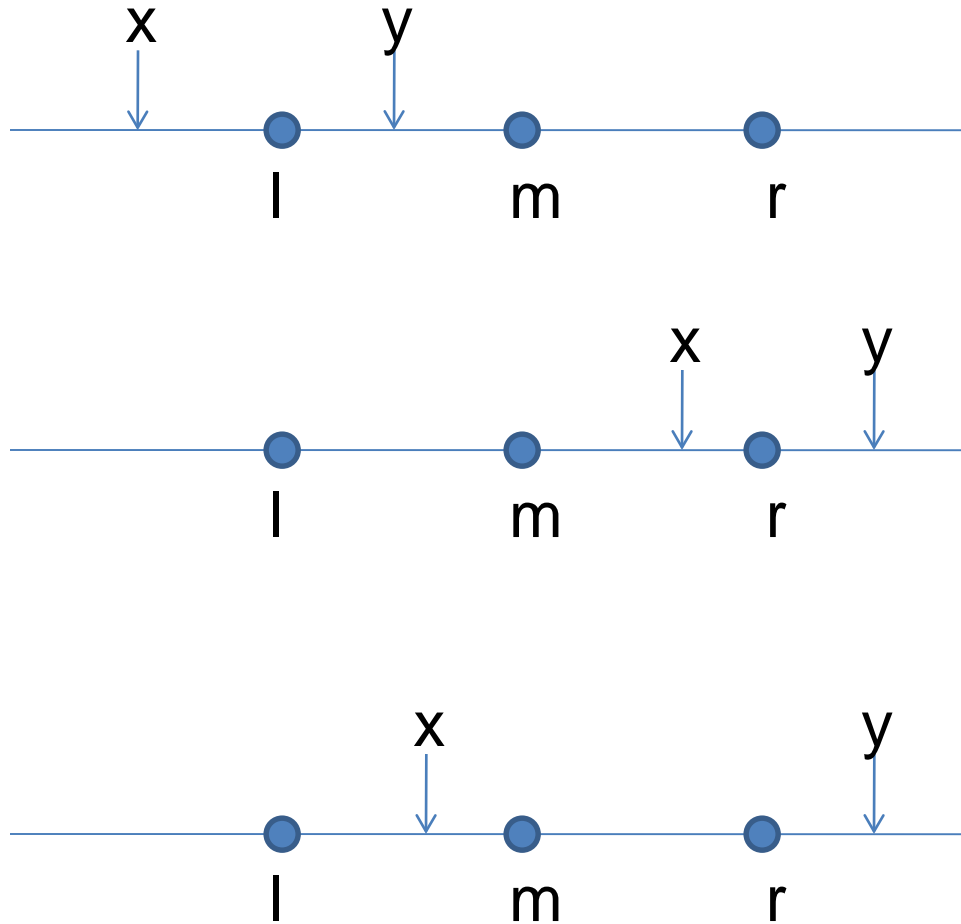
# Corollary of the MVT

Given:

- 1)  $n > 2$  (and  $n$  is odd),
- 2) Pairwise majority rule voting,
- 3) alternatives are on a single dimension,
- 4) preferences are single peaked, and
- 5) preference are symmetric.

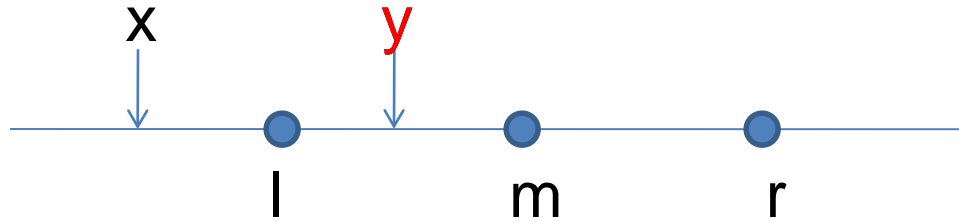
1. **Corollary to the MVT:** alternatives closer to the median voter will defeat alternatives farther from the median voter under pairwise majority rule.

# Sketch of proof of the Corollary

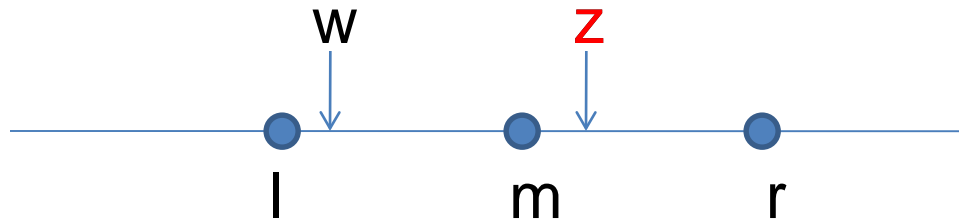
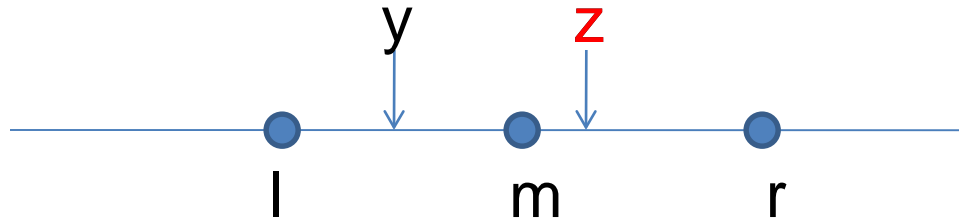
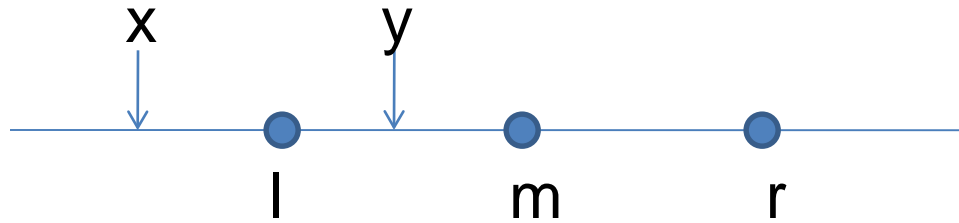


Note: the alternative closer to the median gets the median's vote and half the voters to one side. That's why the closer alternative always wins.

Because of the corollary, alternatives will be drawn toward the median.



Because of the corollary, alternatives will be drawn toward the median.



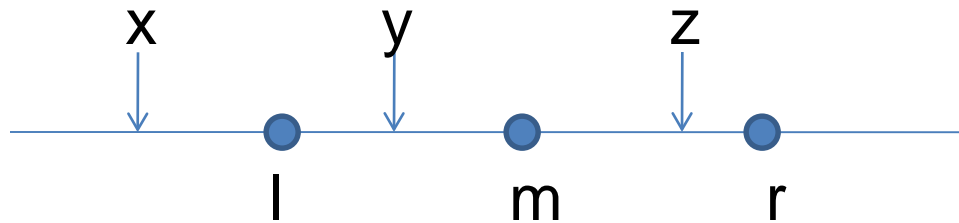
# Other Results

Given:

- 1)  $n > 2$  (and  $n$  is odd),
  - 2) pairwise majority rule voting,
  - 3) alternatives are on a single dimension,
  - 4) preferences are single peaked,
  - 5) and preferences are **symmetric**.
2. The social preference ordering formed by majority rule is the same as the median voter's preference ordering.
  3. Social preferences created by majority rule are transitive (as are the median voter's preferences).



# Example



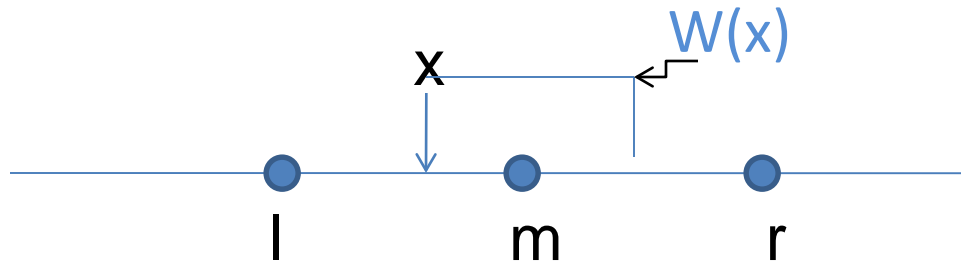
**What is  $m$ 's preference order for  $x$ ,  $y$ ,  $z$ ?**

**What wins under pairwise majority rule:  $x$  vs.  $y$ ,  $y$  vs.  $z$ ,  $x$  vs.  $z$ ?**

**Social preferences are:**

# Win sets

Let the majority rule “win set” of  $x$  be the set of alternatives that a majority prefers to  $x$ :  $W(x) = \{y \mid yPx\}$



The win set is the set of alternatives that can pass – which helps us study status quos that are not in equilibrium.

# Application 1: Location of the Capital

On July 28, 1788 Congress began to vote on the location of the capital. Can you guess what location they agreed upon? D.C. was not an option.

Original 13 States



NH  
MA  
CT  
RI  
NY  
NJ  
PA  
DE  
MD  
VA  
NC  
SC  
GA

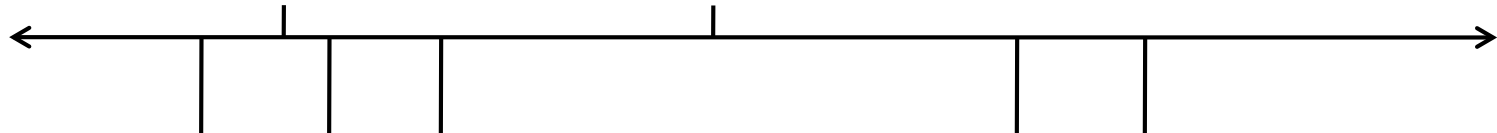
Status Quo: New York City

# Application 2: Two Candidate Election

Who is at the  
median?

Who is closer  
to the median?

Candidates:



Voters: A B C

D E

Obama:

A, B, C

Romney:

D, E

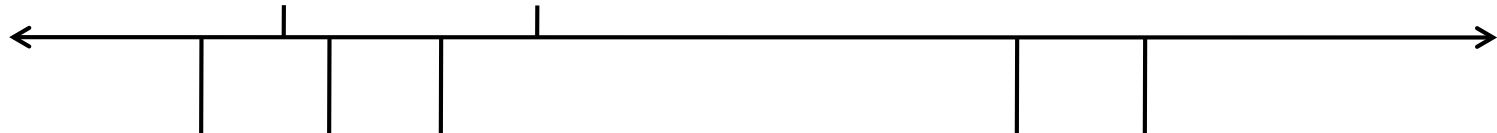
Obama wins, because he gets C's vote and half of the others. What should Romney do?

# Application 2: Two Candidate Election

Who is at the  
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Who is closer  
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Candidates:



Voters: A B C D E

Obama:

A, B

Romney:

C, D, E

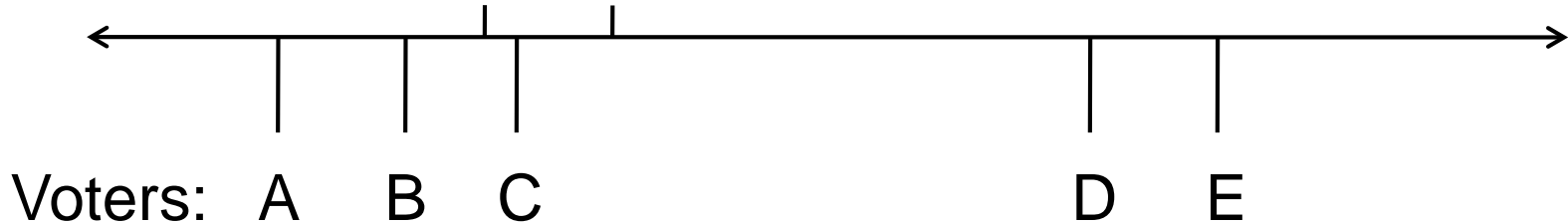
Romney wins, because he gets C's vote and half of the others. What should Obama do?

# Application 2: Two Candidate Election

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Obama:

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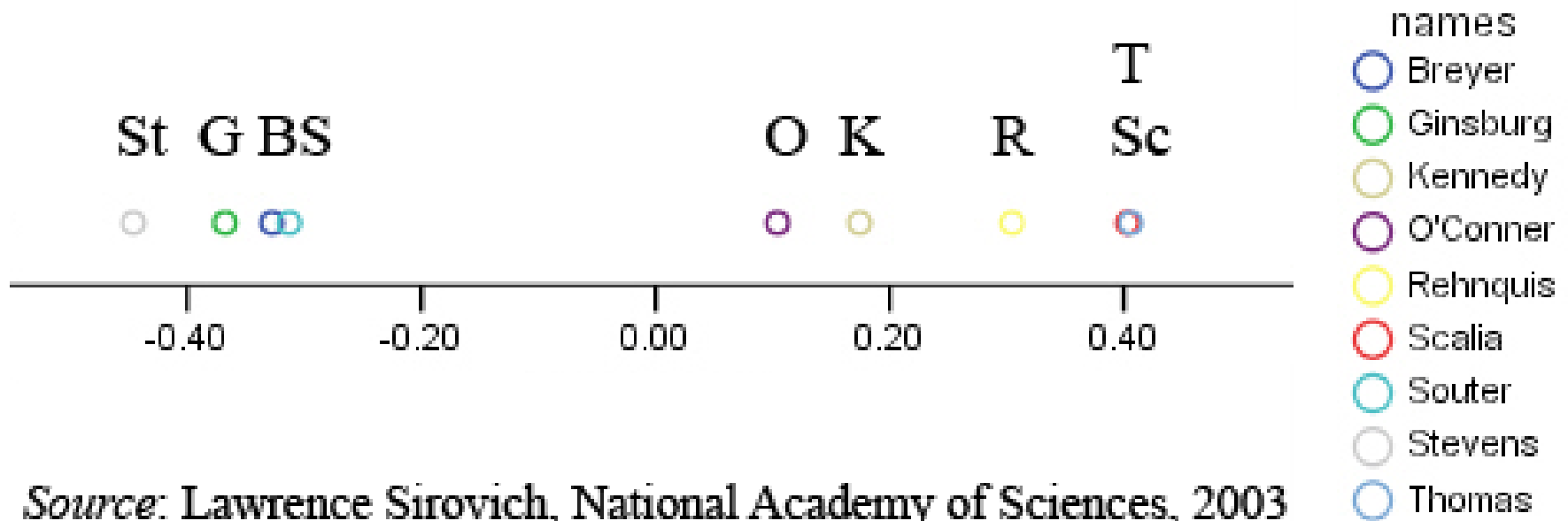
Romney:

D, E

Obama wins, because he gets C's vote and half of the others.

# Application 3: Supreme Court

- US Supreme Court (1995-2002)



*Source:* Lawrence Sirovich, National Academy of Sciences, 2003

- What happened when we replaced Rehnquist with Roberts?
- What happened when we replaced O'Connor with Alito?
- What happened when we replaced Souter with Sotomayor?
- What happened when we replaced Stevens with Kagan?

# Application 4: Legislative chambers and committees

- Assume an odd number of members on the floor and in committee.
- Preferences are single-peaked and symmetric.
- The legislature and committee each use simple majority rule.

## Implication

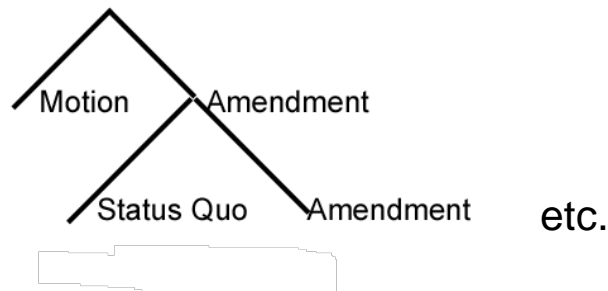
- Median voter results imply the analysis can be simplified by considering only the median of the legislature and the median of the committee.



# Application 4: Legislative chambers and committees

## The effect of open vs. closed rules in Congress

- Open: allows any amendment germane to a bill to be proposed on the floor.



- b. Closed Rule: prohibits amendments on the floor.

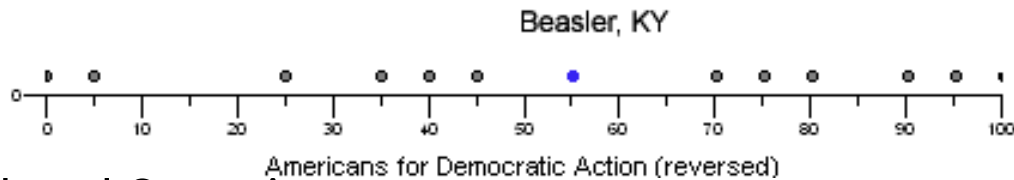


**Membership on House Agricultural Subcommittees Dealing with Tobacco**

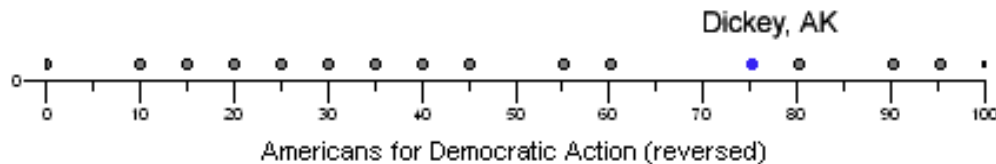
<b>Subcommittee on Tobacco, 1973</b>	<b>Subcommittee on Tobacco and Peanuts, 1983</b>	<b>Subcommittee on Specialty Crops &amp; Natural Resources, 1993</b>
Stubblefield, D-KY	Rose, D-NC	Rose, D-NC
Jones, D-NC	Jones, D-NC	Baessler, D-KY
Mathis, D-GA	Hatcher, D-GA	Bishop, D-GA
Rose, D-NC	Thomas, D-GA	Brown, D-CA
Litton, D-MO	Whitley, D-NC	Condit, D-CA
Mizell, R-NC	Tallon, D-SC	Clayton, D-NC
Wampler, R-VA	English, D-OK	Thurman, D-FL
Madigan, R-IL	Stenholm, D-TX	Minge D-MN
Young, R-SC	Hopkins, R-KY	Inslee, D-WA
	Roberts, R-KS	Pomeroy, D-ND
	Skeen, R-NM	English, D-OK
	Franklin, R-MS	Senholm, D-TX
		Peterson, D-MN
		Lewis, R-FL
		Emerson, R-MO
		Doolittle, R-CA
		Kingston, R-GA
		Goodlatte, R-VA
		Dickey, R-AK
		Pombo, R-CA

Source: Loomis, *The Contemporary Congress*, Table 5-2.

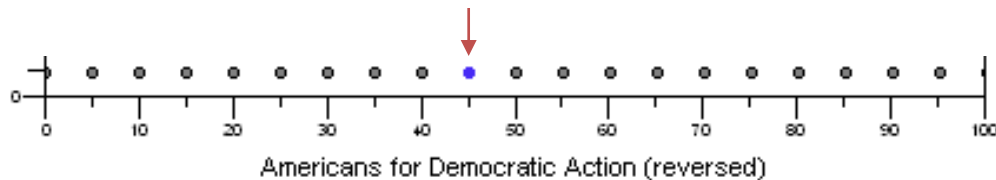
- Subcommittee on Specialty Crops



- Agricultural Committee



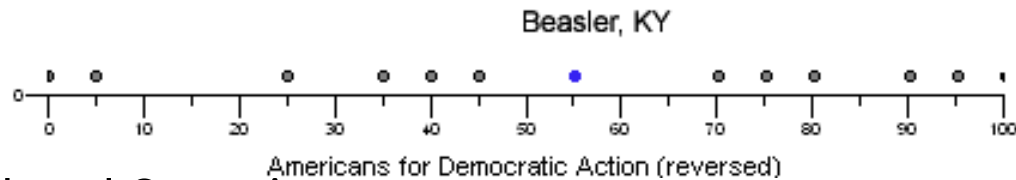
- House Floor



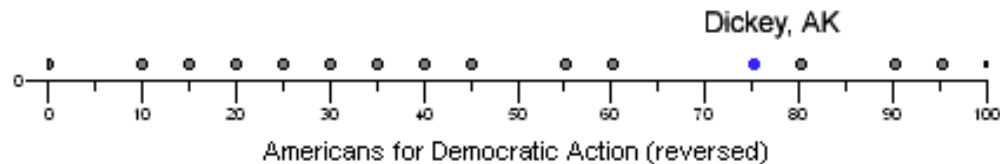
If an open rule was allowed on the floor, what do you predict would be the outcome of a bill coming out of this committee after it was amended by the floor?

- a. The floor median.

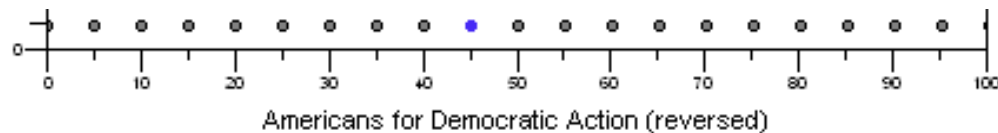
- Subcommittee on Specialty Crops



- Agricultural Committee

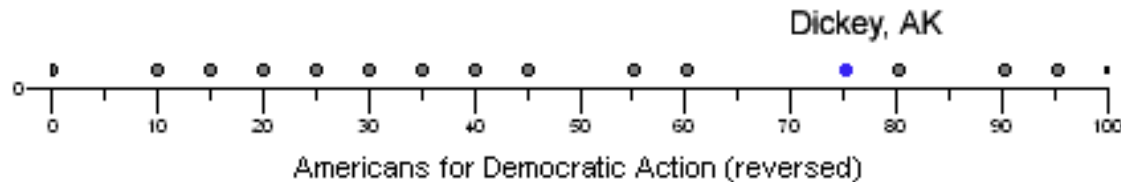


- House Floor

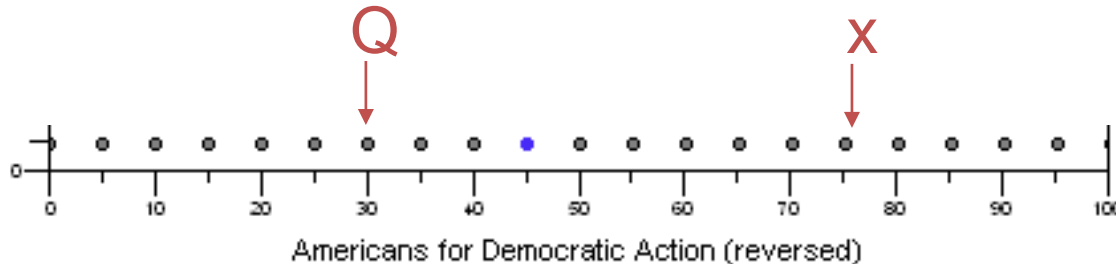


What if the bill was voted on using a closed rule?

– Agricultural Committee



2. House Floor

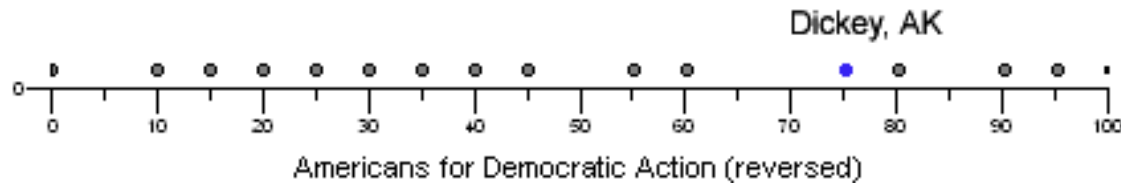


To make this simple, I'm going to get rid of the sub-committee for the moment.

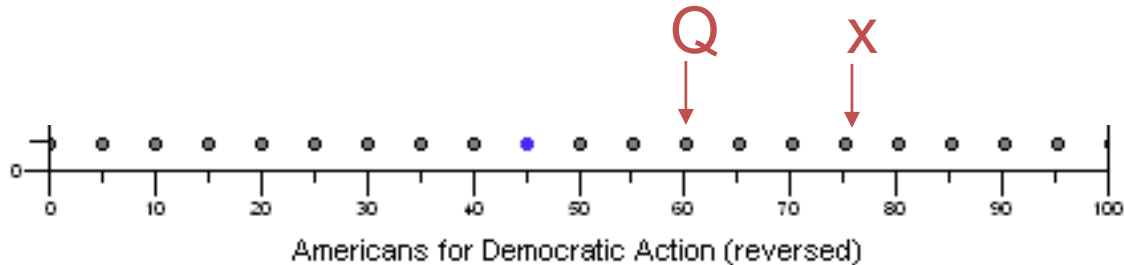
Now suppose the status quo,  $Q = 30$ , the committee proposes  $x = 75$ , and the floor votes under a closed rule. What should be the outcome?

- a. Q wins. Under a closed rule, the choice is between  $x$  and  $Q$ . Since  $Q$  is closer to the floor median,  $Q$  wins.

– Agricultural Committee



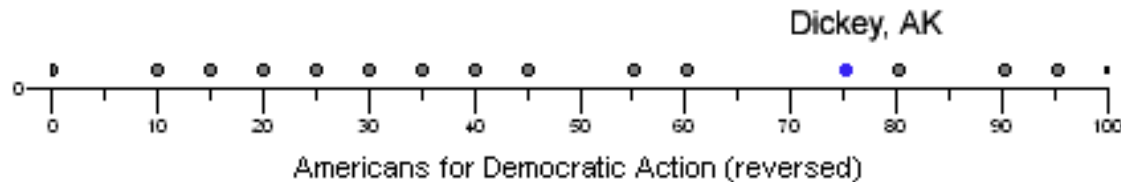
2. House Floor



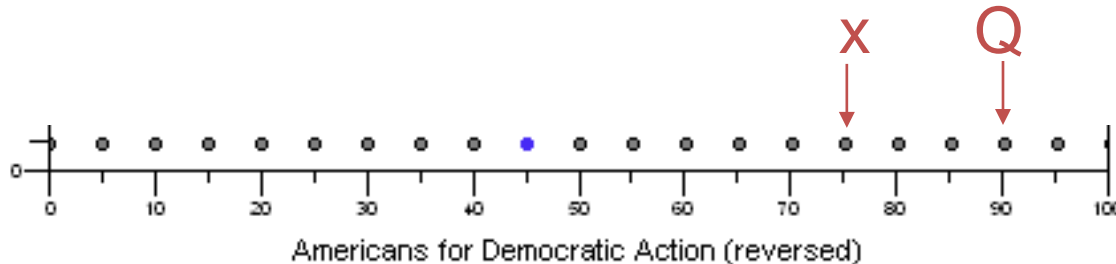
Now suppose the status quo,  $Q = 60$ , the committee proposes  $x = 75$ , and the floor votes under a closed rule. What should be the outcome?

- a. Q wins. Under a closed rule, the choice is between  $x$  and  $Q$ . Since  $Q$  is closer to the floor median,  $Q$  wins.

– Agricultural Committee



2. House Floor

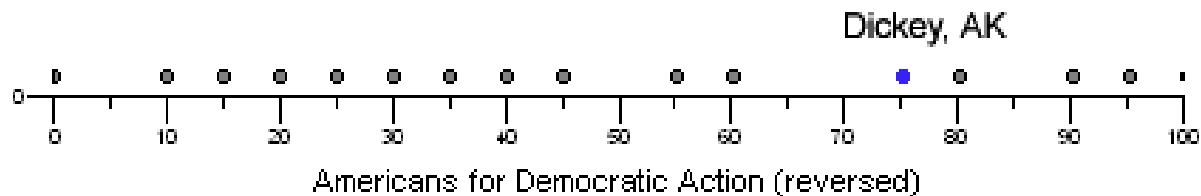


Now suppose the status quo,  $Q = 90$ , the committee proposes  $x = 75$ , and the floor votes under a closed rule. What should be the outcome?

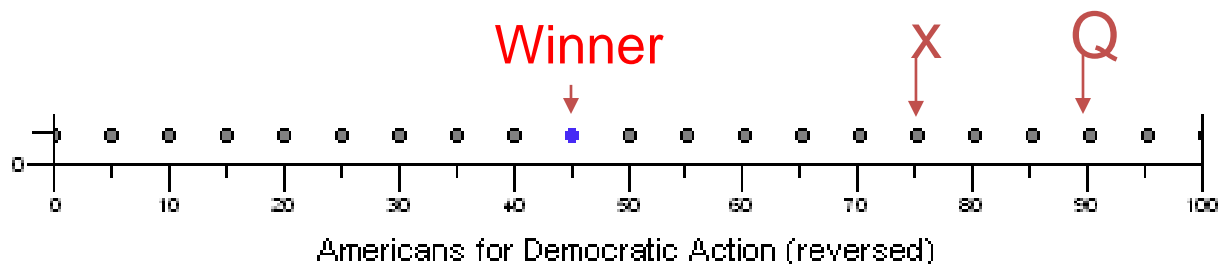
- $x = 75$  wins. Under a closed rule, the choice is between  $x$  and  $Q$ . Since  $x$  is closer to the floor median,  $x$  wins.

What if the vote was under an open rule?

## – Agricultural Committee



## 2. House Floor



Now suppose the status quo,  $Q = 90$ , the committee proposes  $x = 75$ , and the floor votes under a closed rule. What should be the outcome?

- $x = 75$  wins. Under a closed rule, the choice is between  $x$  and  $Q$ . Since  $x$  is closer to the floor median,  $x$  wins.

What if the vote was under an open rule?



# Summary

- The Median Voter Theorem explains why alternatives cannot leave the median position.
- The corollary to the Median Voter Theorem explains why alternatives are drawn to the median position.
- Under an open rule, the House floor median might be the expected outcome.
  - This makes the floor powerful, not the committee.
- Under a closed rule, either the status quo or the committee proposal should win (depending upon which alternative is closer to the floor median).
  - This makes the committee more powerful than under an open rule.