MULTIPLE DIMENSIONS 2: SPE, UNCOVERED SET, BANKS SET

Last Week on Multiple Dimensions

- If the Plott conditions are not met,…
  - the majority rule core is empty (i.e., no equilibrium).
  - Majority rule creates an *intransitive* order for the entire set of alternatives.
  - An agenda setter could take us anywhere.
- Today (moves toward stability)
  - Pareto set.
  - Rational agenda setting in SPE.
  - Uncovered Set.
  - Banks Set.

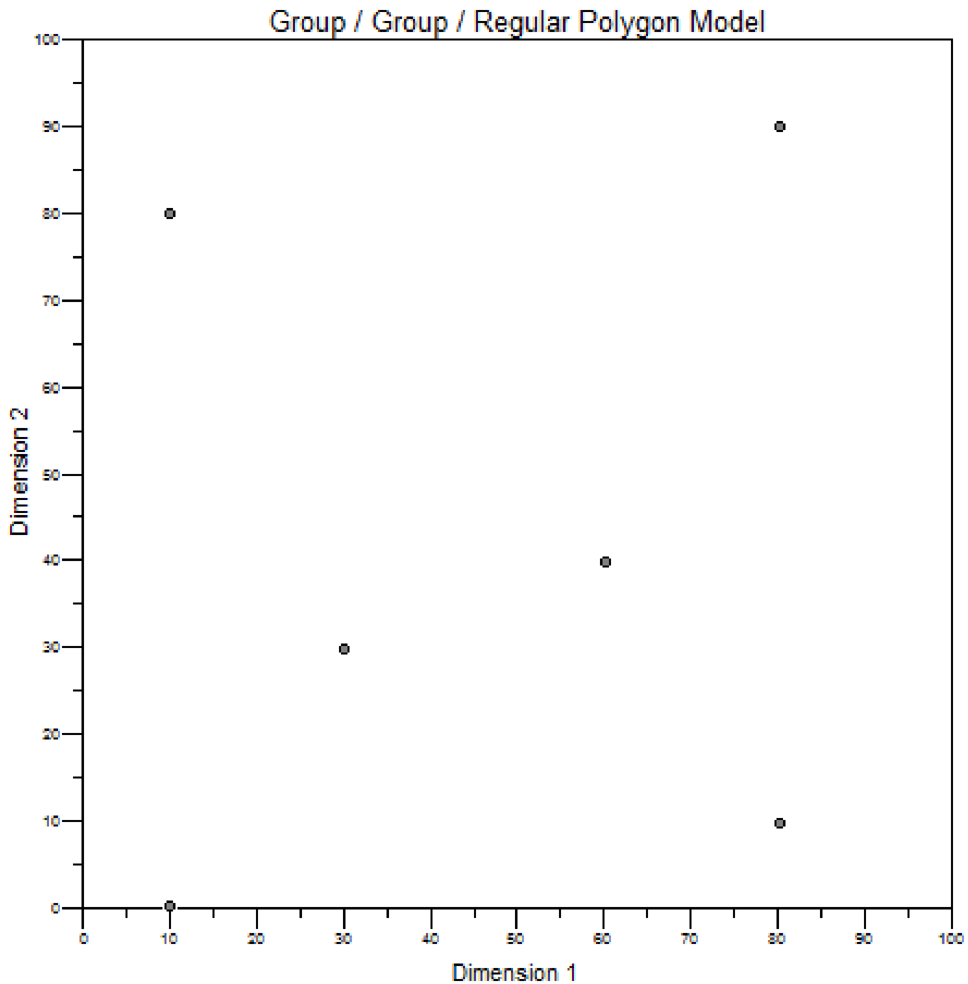
Pareto Criterion

Definition: For any two alternatives *x* and *y*, *x* is Pareto preferred to *y* if and only if it makes at least one individual better off than *y* and no individual worse off than *y*.

- If *x* adheres to this definition with respect to *y*, we will say that *x* is **Pareto preferred** (or Pareto superior) to *y*.
- Economists equate Pareto improvements (i.e., Pareto superior moves) with improvements in efficiency because it means less well-being is wasted.

Pareto Criterion

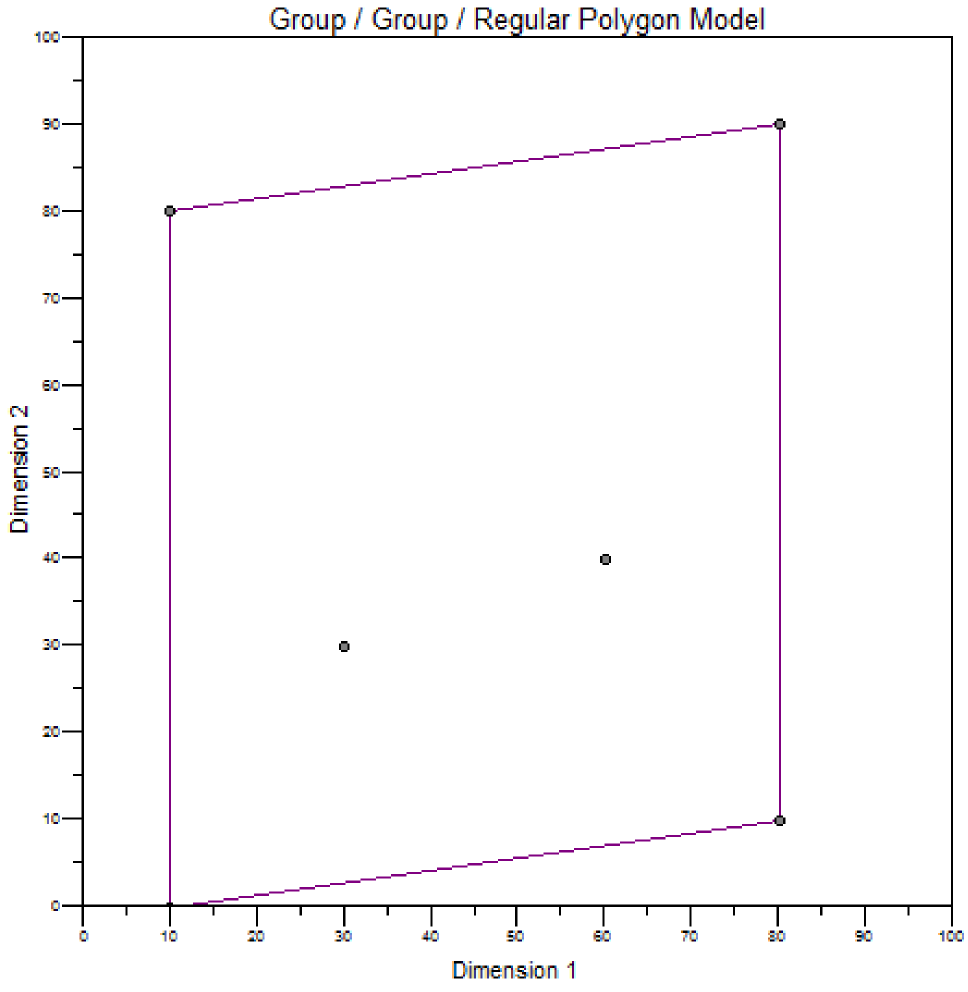
6 ideal points.



Pareto Criterion

6 ideal points.

Convex hull
(purple polygon)



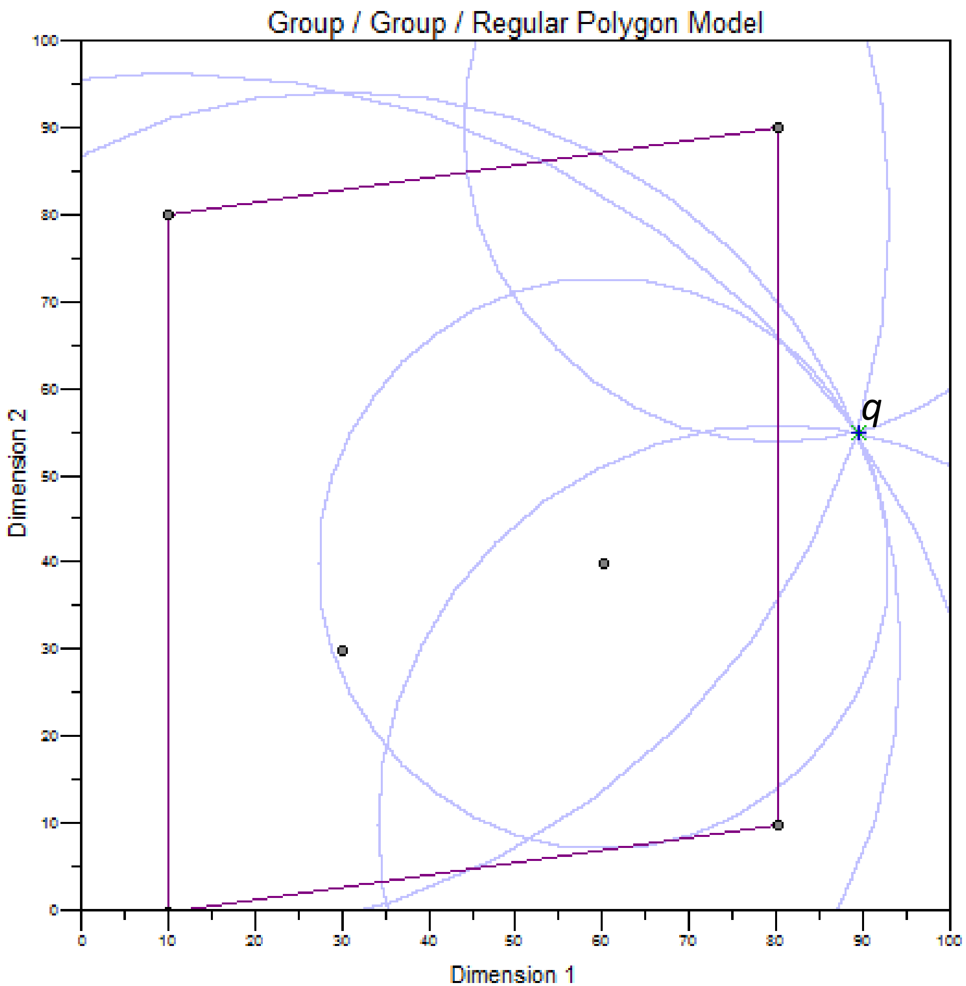
Pareto Criterion

6 ideal points.

Convex hull (purple polygon)

Status quo, *q*, with indifference curves drawn through it.

What points are Pareto preferred to *q*?



Pareto Criterion

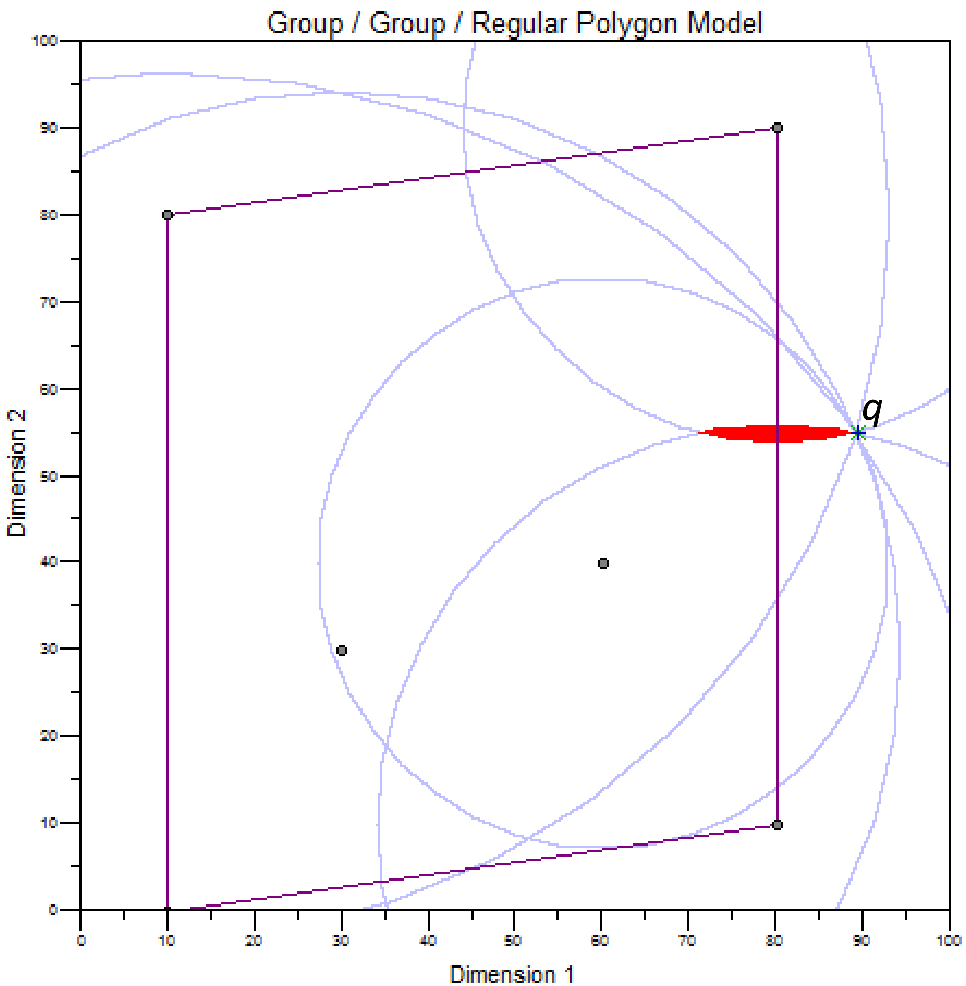
6 ideal points.

Convex hull (purple polygon)

Status quo, *q*, with indifference curves drawn through it.

What points are Pareto preferred to *q*?

Red petal contains all points Pareto preferred to *q*.



More generally,

1- For all points outside the convex hull, there is always a non-empty set of Pareto preferred points (at least part of which is in the hull).

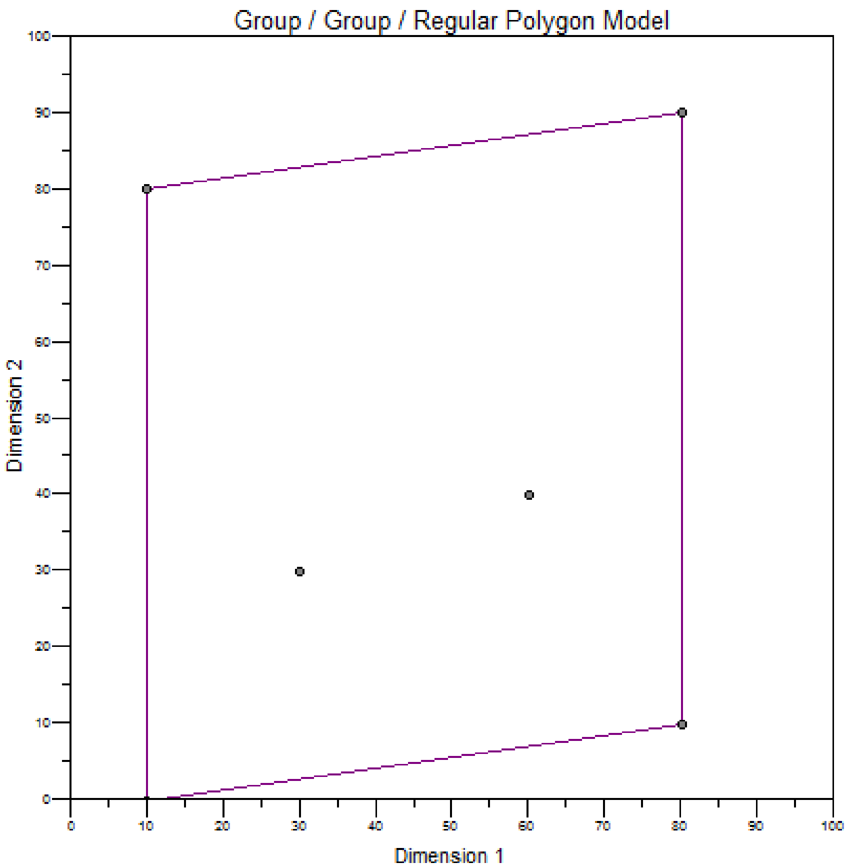
2- For all points inside the convex hull, the set of Pareto preferred points is empty.

Pareto Optimality

Definition: Alternative *x* is Pareto optimal, if there does not exist an alternative *y* that is Pareto preferred to *x*.

The Convex hull (inclusive) is the **Pareto set**, i.e., the exhaustive set of Pareto optimal points.

Think of Pareto optimality as an efficiency condition. We would not want to chose a Pareto suboptimal point (i.e., a point outside the convex hull) because there is always a point inside the hull that makes everyone better off. We should chose something that makes everyone better off instead.



Dougherty-Edward Theorem

Assume:

1. $R \geq 1$ finite rounds of voting,
2. a voter is designated proposer in the last round (a variety of proposers and proposal processes can be used in earlier rounds),
3. proposals are strategic in the last round,
4. individuals vote strategically (or sincerely), and
5. complete information.

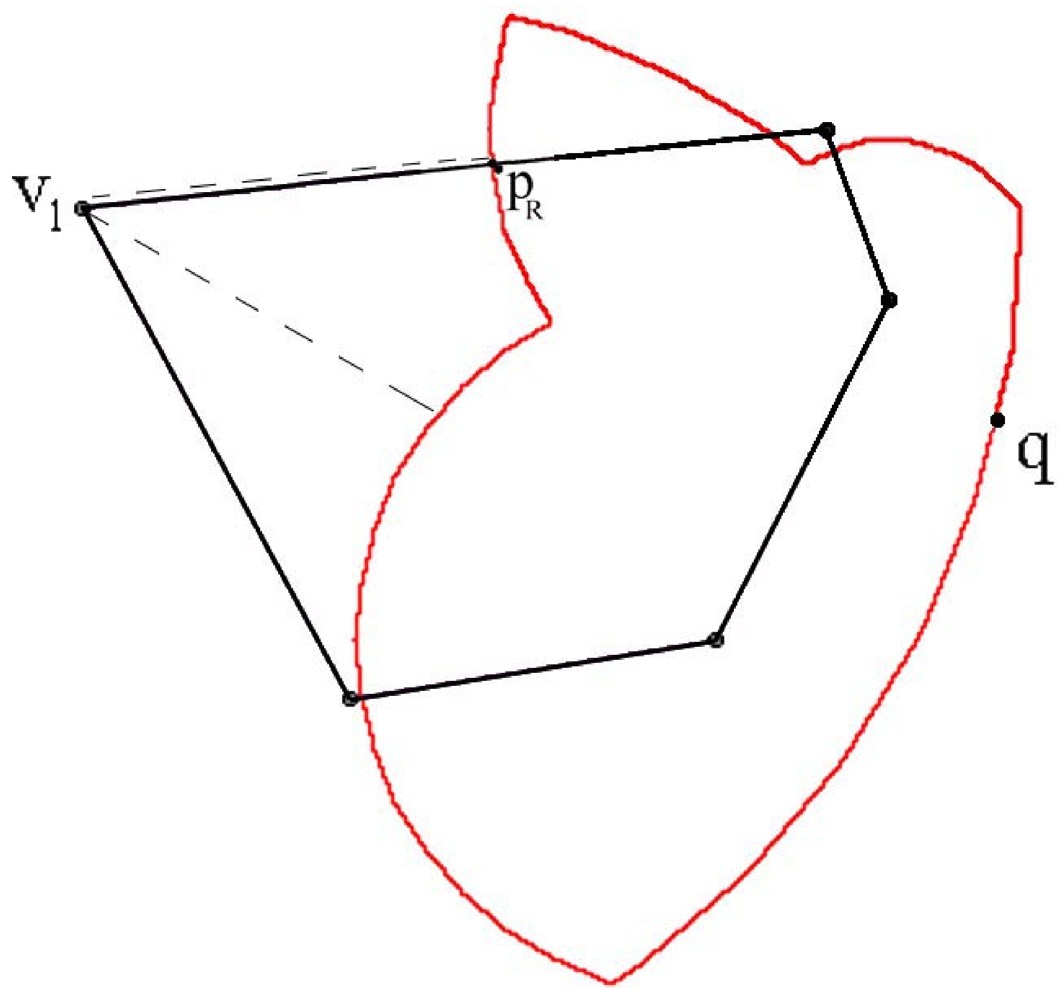
Dougherty-Edward Theorem

Theorem. Denote by q_R the status quo in the final round R. Suppose there exists a point $z \in W(q_R)$ of minimal distance to the proposer (including empty $W(q_R)$). Then given assumptions 1-5, a group using majority rule will select a Pareto optimal outcome in subgame perfect equilibrium (SPE).

Remarks

- Uses a different equilibrium concept than the core.
- The theorem shows that in finite rounds of play, a rational agenda setter will get us to an outcome in the Pareto set. He/she will not let us wonder anywhere.
- Differs from McKelvey by 1) modeling the proposal process and 2) applying SPE rather than a core.

Dougherty-Edward Theorem



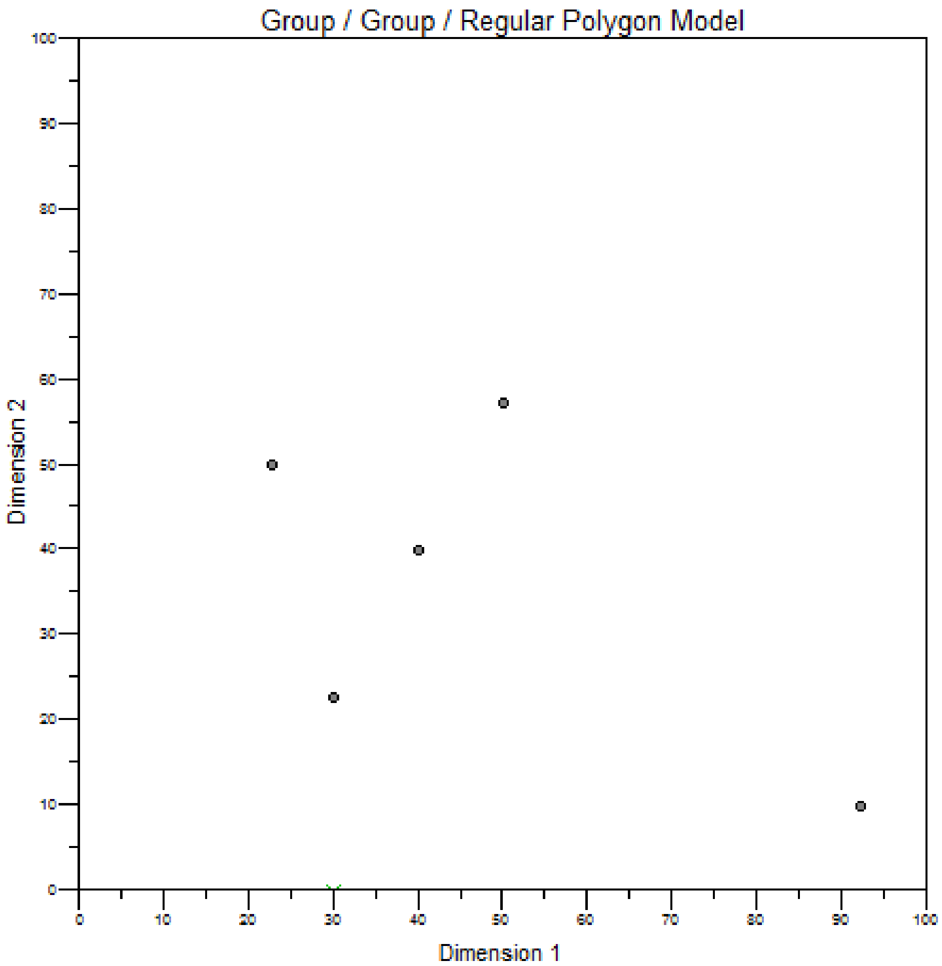
Dougherty-Edward Theorem

Additional Remarks

- The same is true for all k-majority rules by the way, suggesting that unanimity rule may not be particularly adept at selecting Pareto optimal outcomes (as previously thought).
- The theorem is very flexible about the type of amendment process and the number of alternatives in that amendment process.

Extra Credit Results

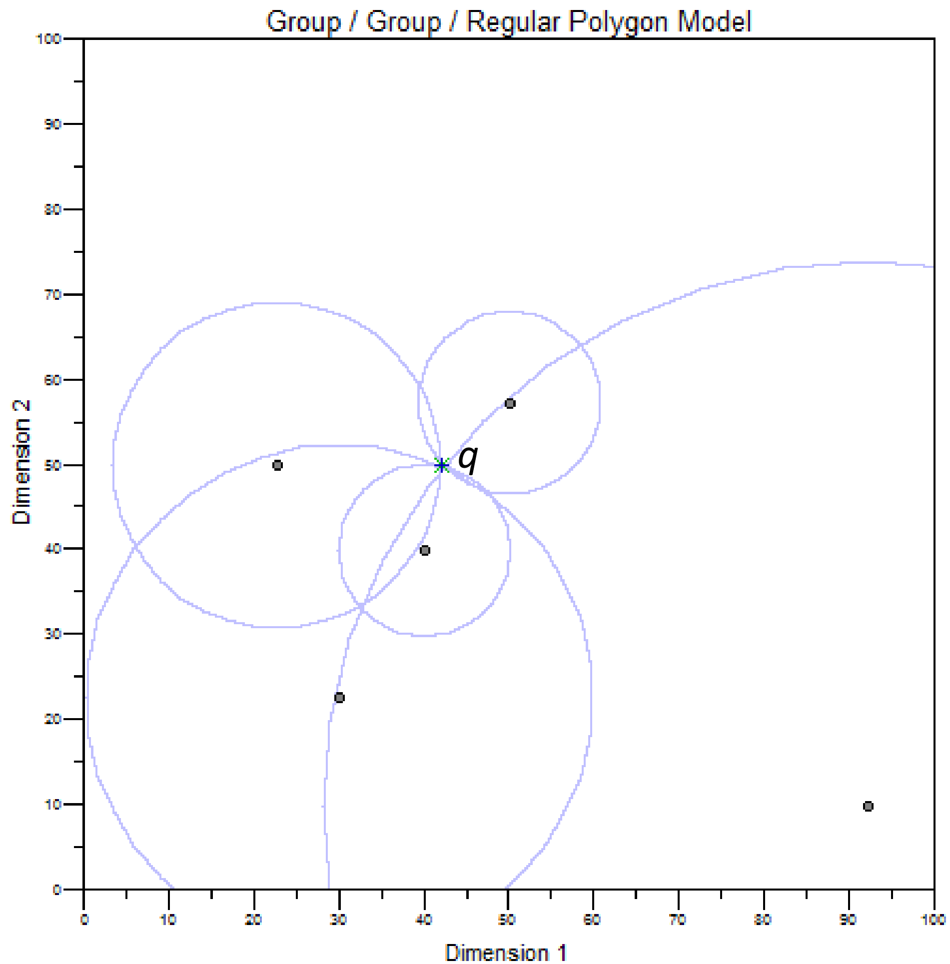
Ideal points from *your* extra credit game, rescaled.



Extra Credit Results

Ideal points from *your* extra credit game, rescaled.

This is where you ended, with some indifference curves.



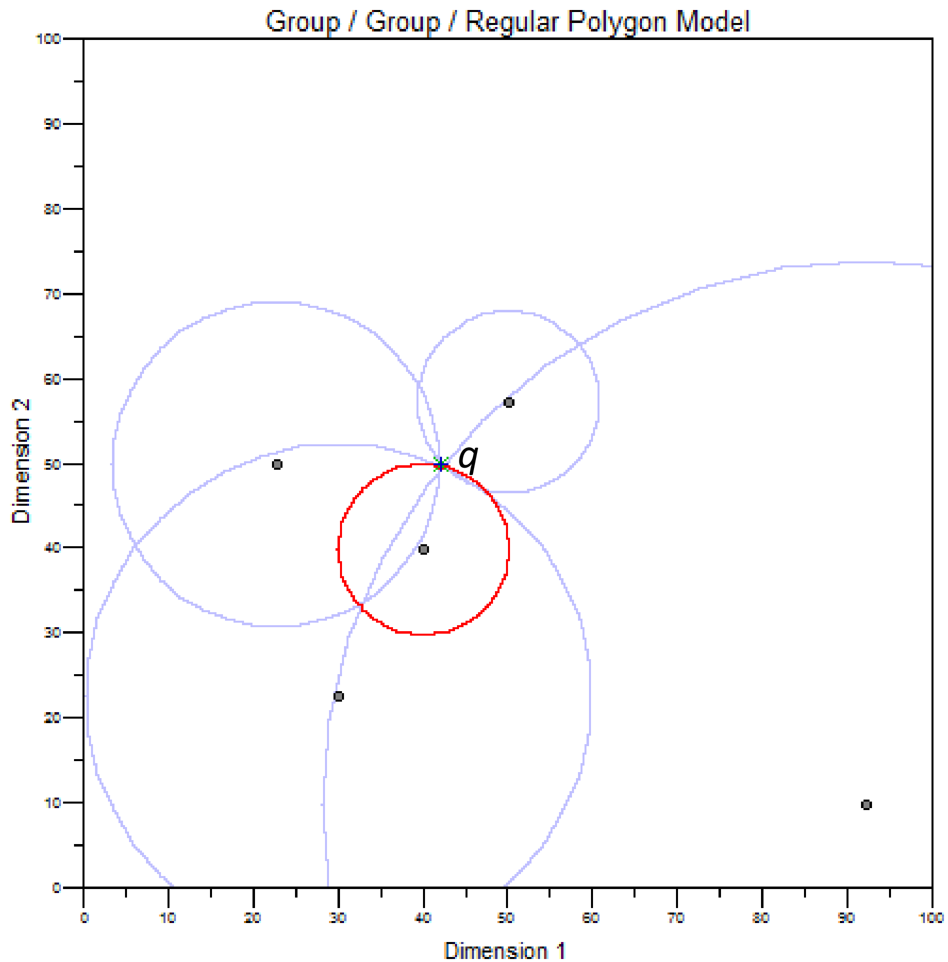
Extra Credit Results

Ideal points from your extra credit game, rescaled.

This is where you ended, with some indifference curves.

The point can be beaten by everything in red. Hence, it is not in the core.

But it was in the Pareto set.



CyberSenate for other cases.

Covered Alternatives & The Uncovered Set

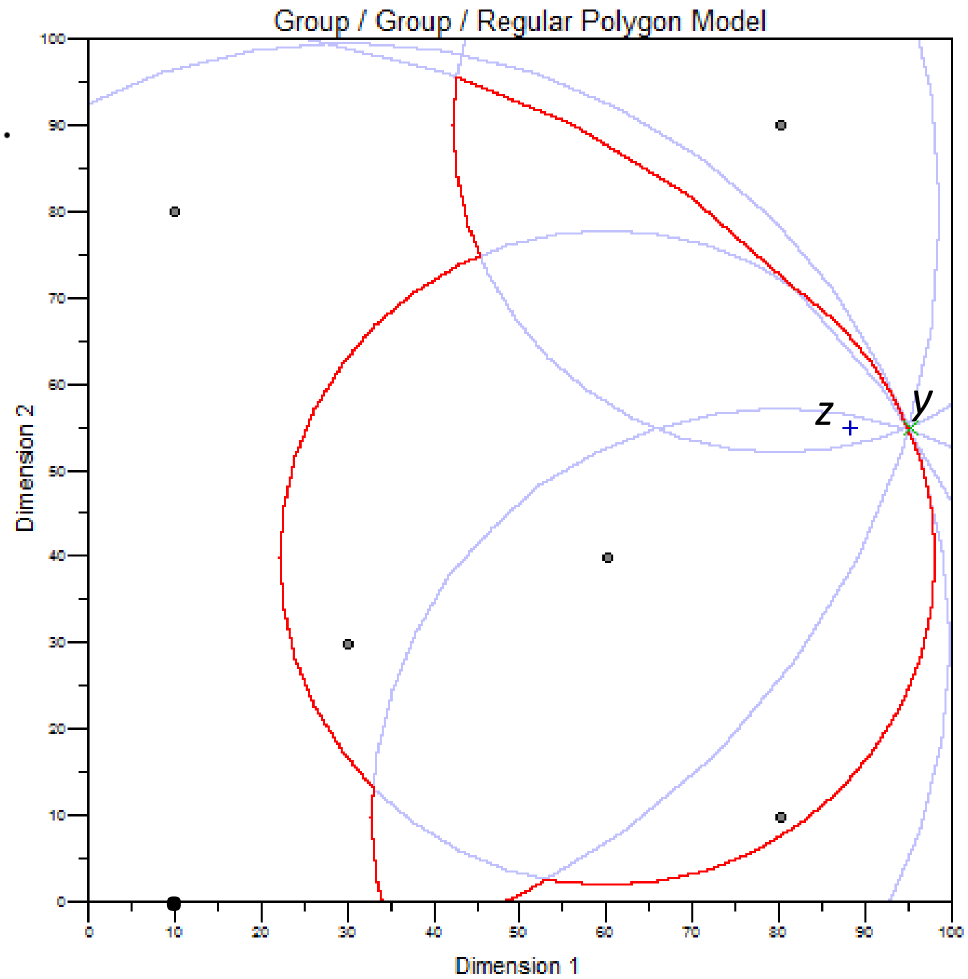
Definition

- Alternative *y* is **covered** by an alternative, *z*, if *z* is majority preferred to *y* and if, for every *x* to which *y* is majority preferred to *x*, *z* is also majority preferred to *x*.
  - Put differently, *y* is covered by *z* if the win set of *y* contains the win set of *z*.
  - Note: this is not about the superiority of *z*; it is about the inferiority of *y*.

Y is covered by Z

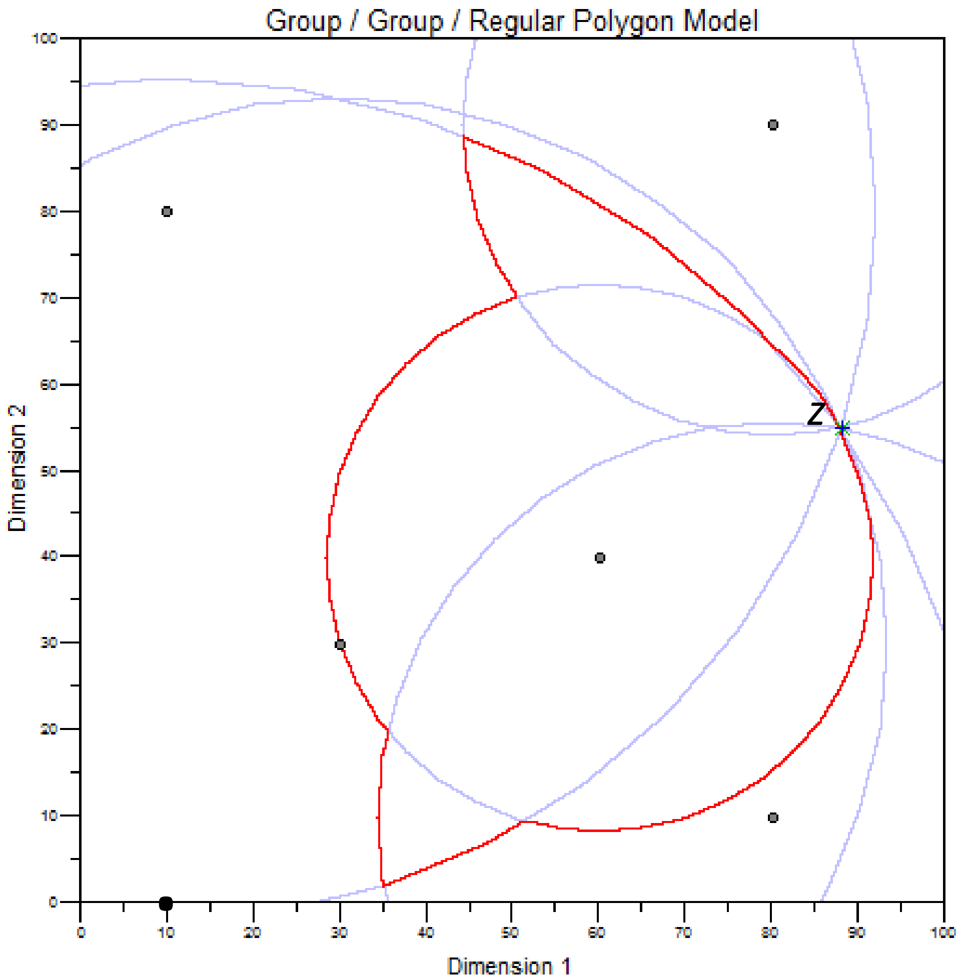
Win set of *y*.

Note, $z \in W(y)$.



Y is covered by Z

Win set of *z*.

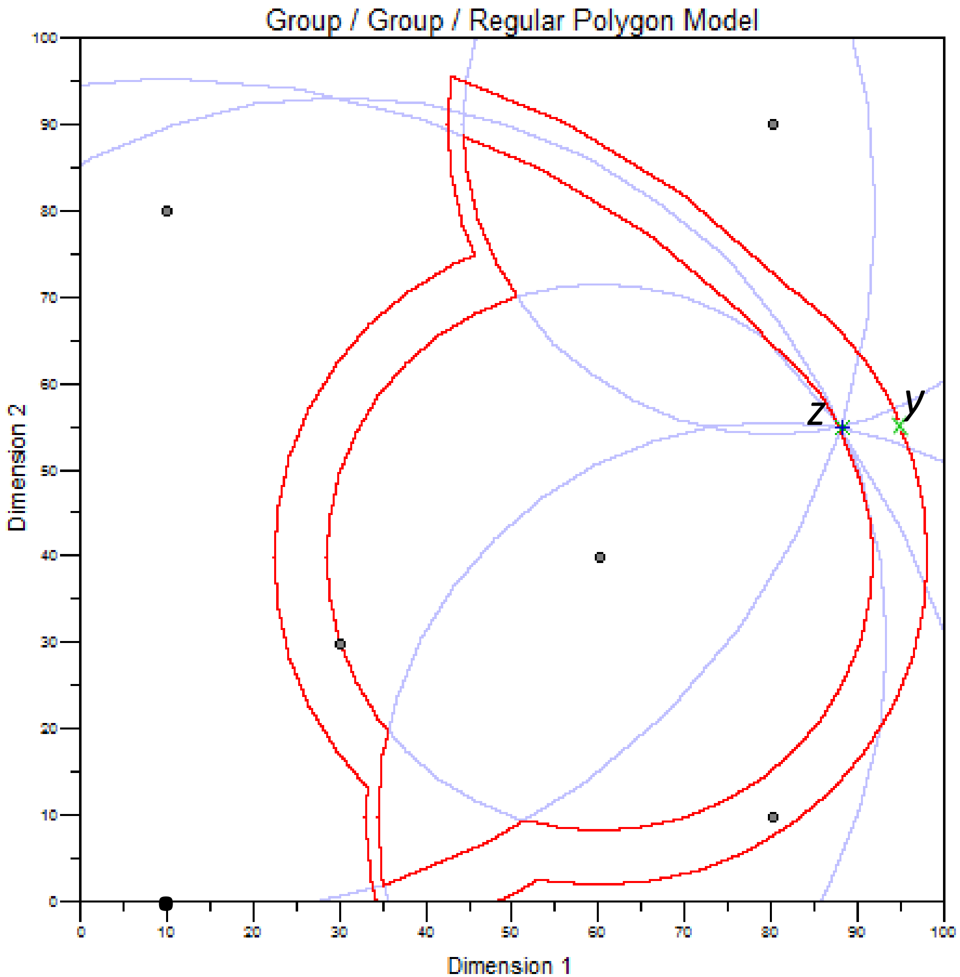


Y is covered by Z

Note, $W(z) \subset W(y)$

...Hence, z covers y.

Some would argue that an agenda game would not stop at *y* because *y* is covered.



Uncovered Set

Definition

- The **uncovered set** (UC set) is the set of uncovered alternatives -- i.e. those alternatives that do not have another point(s) covering them.

Relationship to Pareto Set

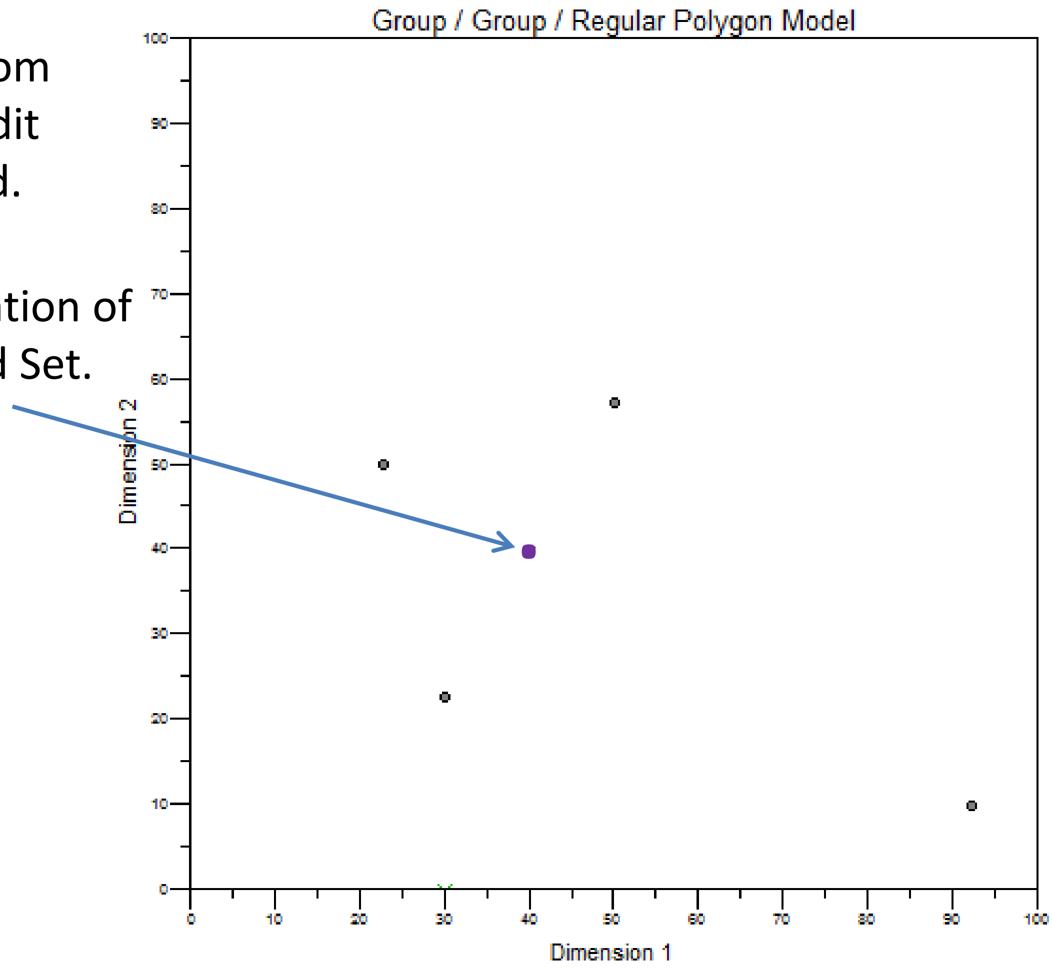
- For all points outside the convex hull, such as *y*, there must be exist a point, *z'*, in the hull that is Pareto preferred to *y*. Hence, $W(z') \subset W(y)$.

- This implies that $\text{UC set} \subseteq \text{Pareto Set.}$
  - It's exact location is hard to calculate, so folks have used grid searches.

Uncovered Set

Ideal points from your extra credit game, rescaled.

Guess the location of the Uncovered Set.

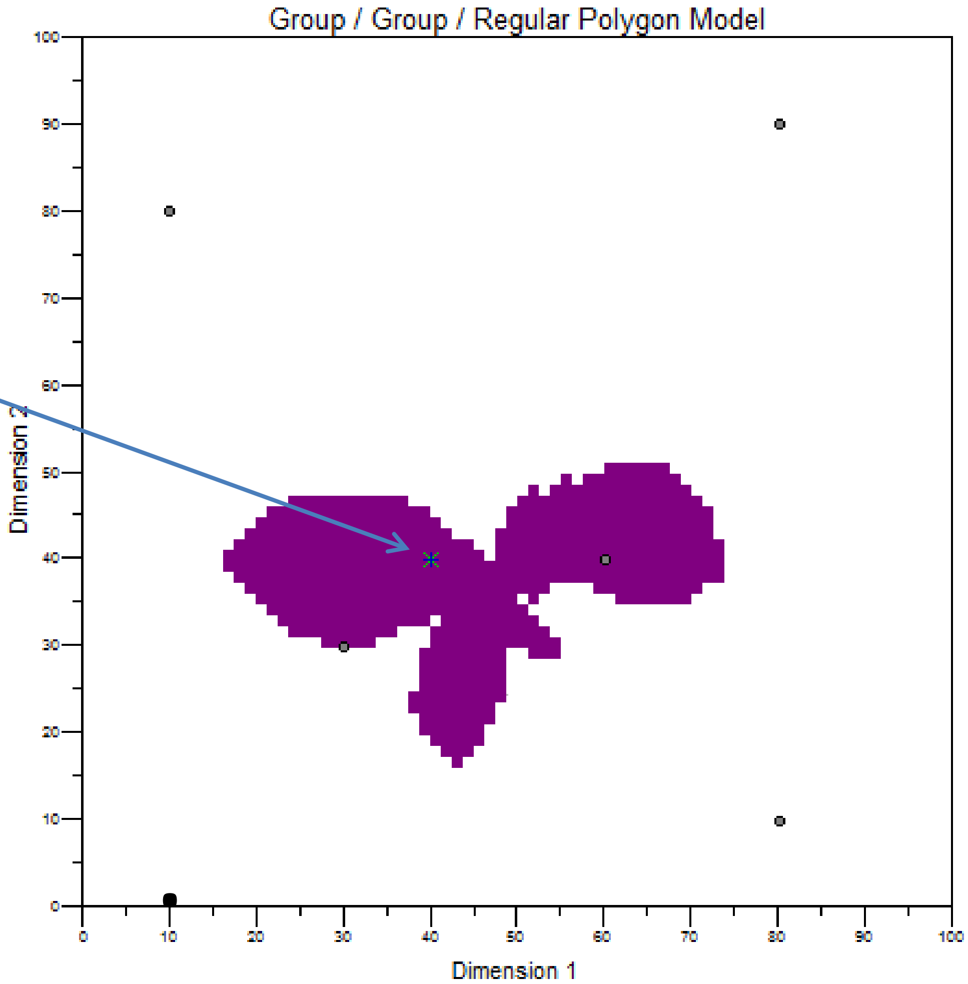


Uncovered Set

UC set in purple.

Note: previous semester ended in the UC set.

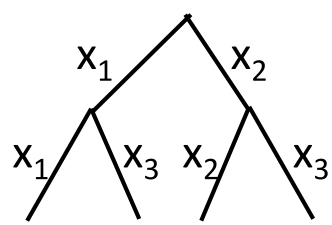
$\text{UC set} \subseteq \text{Pareto Set}$



Uncovered Set

Theorem (Shepsle and Weingast, 1984)

- There exists a finite agenda with y the first element and x the sophisticated agenda equilibrium if and only if y does not cover *x*.
  - Sophisticated agenda equilibrium (SAE). For three alternatives $\{x_1, x_2, x_3\}$.



The SAE is the alternative that wins if everyone votes strategically.

  - This means that as long as the starting point does not cover the sophisticated outcome *x** there exists an agenda that will yield *x**.
    - Note, this does not say that a sophisticated agenda must yield an uncovered outcome. Such agendas might also yield covered outcomes.
    - Furthermore, it does not say that *all* uncovered outcomes will result from a sophisticated agenda equilibrium. Perhaps only a subset will.

Banks Set

Definition

- The **Banks set** (BS) is the set of alternatives resulting from strategic voting in a successive elimination procedure (i.e. SAE).

- **Remarks**
  - Successive Elimination is a specific voting procedure (a tree like the one we just looked at) where an alternative is eliminated if it loses a pairwise contest.
  - The Banks set is the set of strategic outcomes for all possible orderings of the tree.
  - In order for an alternative to lie within the Banks set, the agenda must be defined over the entire space.

Banks Set

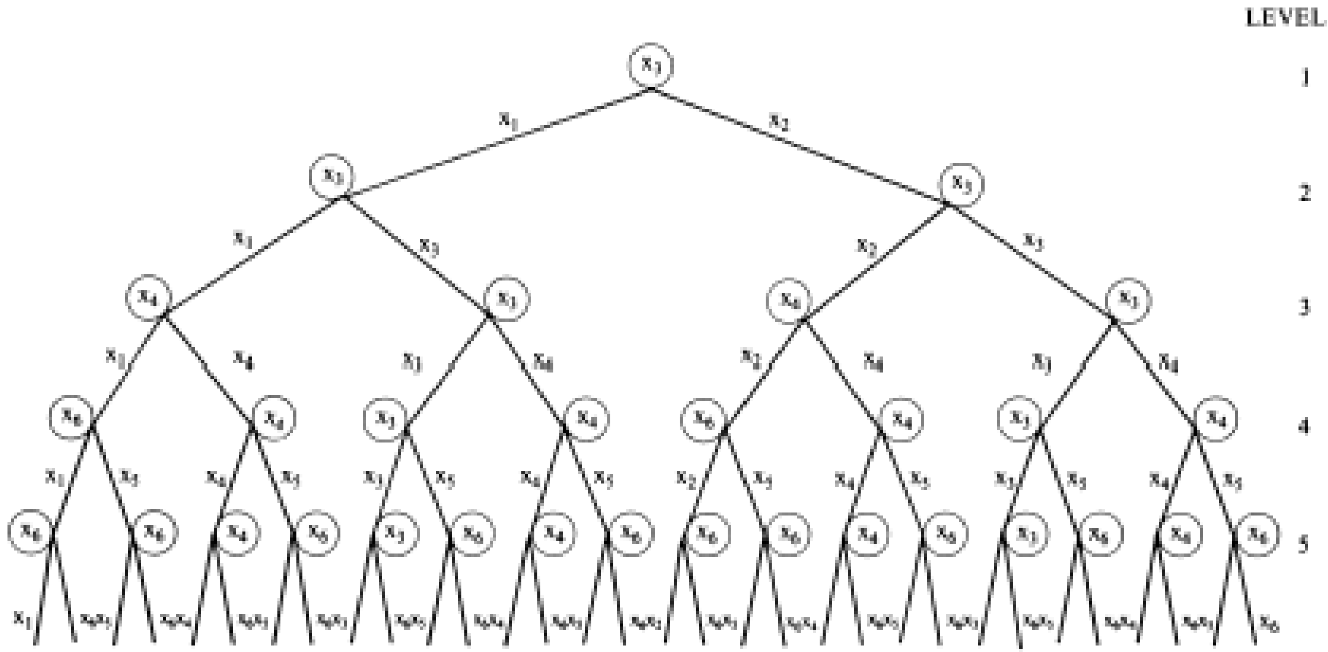


FIGURE 2
Division Scheme for Multistage Game with Five Levels

<u>For six alternatives:</u>

Agenda {x_1, x_2, $x_{3,}$ $x_{4,}$ $x_{5,}$ x_6} will produce a winner.

Agenda {x_1, x_3, $x_{2,}$ $x_{4,}$ $x_{5,}$ x_6} may produce a different winner.

With six alternatives, the B.S. results from 6! agendas.

Banks Set

More Remarks

- With a sufficiently large number of alternatives $BS \subset UC$.
- Note, the Banks set might explain the success of the UC set.

Spatial Location of the Banks Set

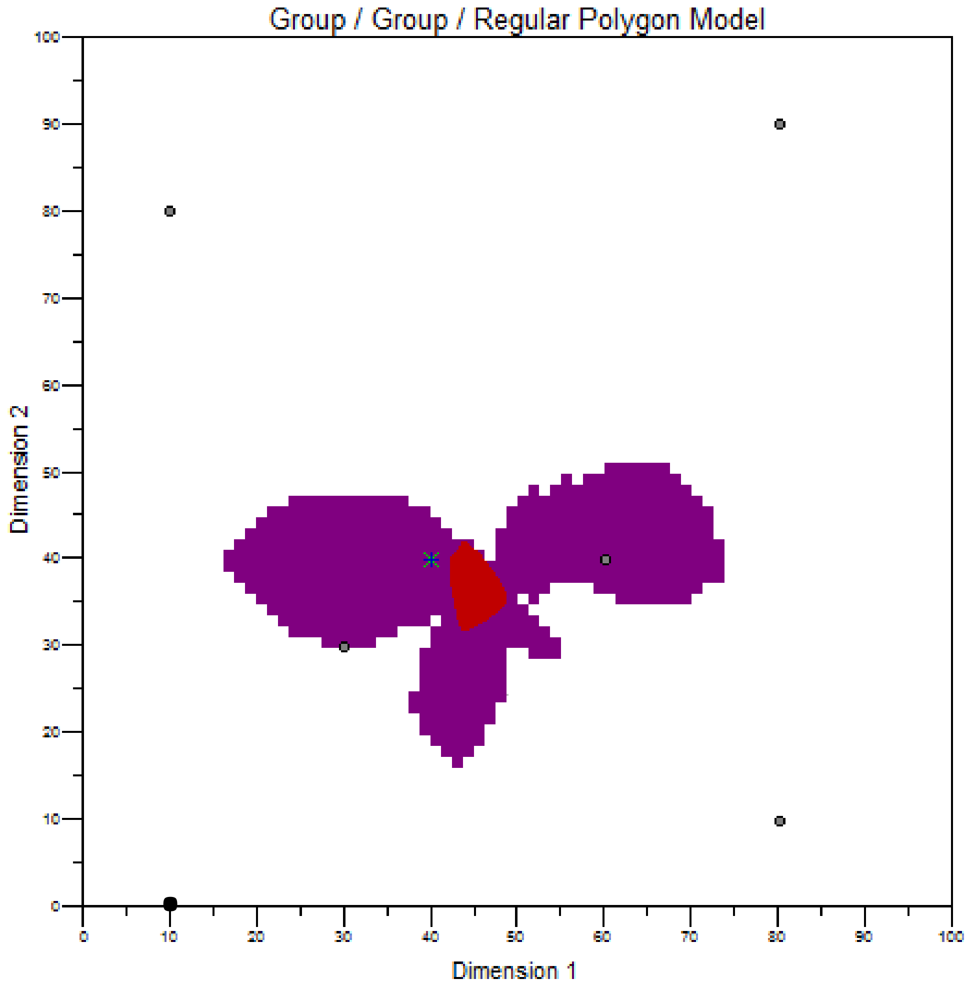
- For more than three voters, the precise location of the Banks set in multi-dimensional models is still unknown (Feld et al. 2013).
  - We do know that the Schattschneider set (SS) is a subset of the Banks set. Hence, $SS \subset BS \subset UC$.
  - The Schattschneider set is the locus of all geometric medians (something easy to calculate).

Schattschneider Set & UC Set

Schattschneider set in red.

Note: your group ended close to the SS.

…but our game was not an elimination procedure.



Experimental Evidence

Bianco, Lynch, Miller, Sened (2008)

- Find strong evidence that both large and small groups end play in the Pareto set.

Dougherty, Moeller, Pitts, & Ragan (2014)

- Subjects are randomly assigned to 32 groups of 7 members each.
- Each group is assigned
  - A voting rule: majority rule or unanimity rule
  - An information condition: complete information or incomplete information.
- Ideal points are matched across the four treatment conditions (so there are 8 unique sets of ideal points).

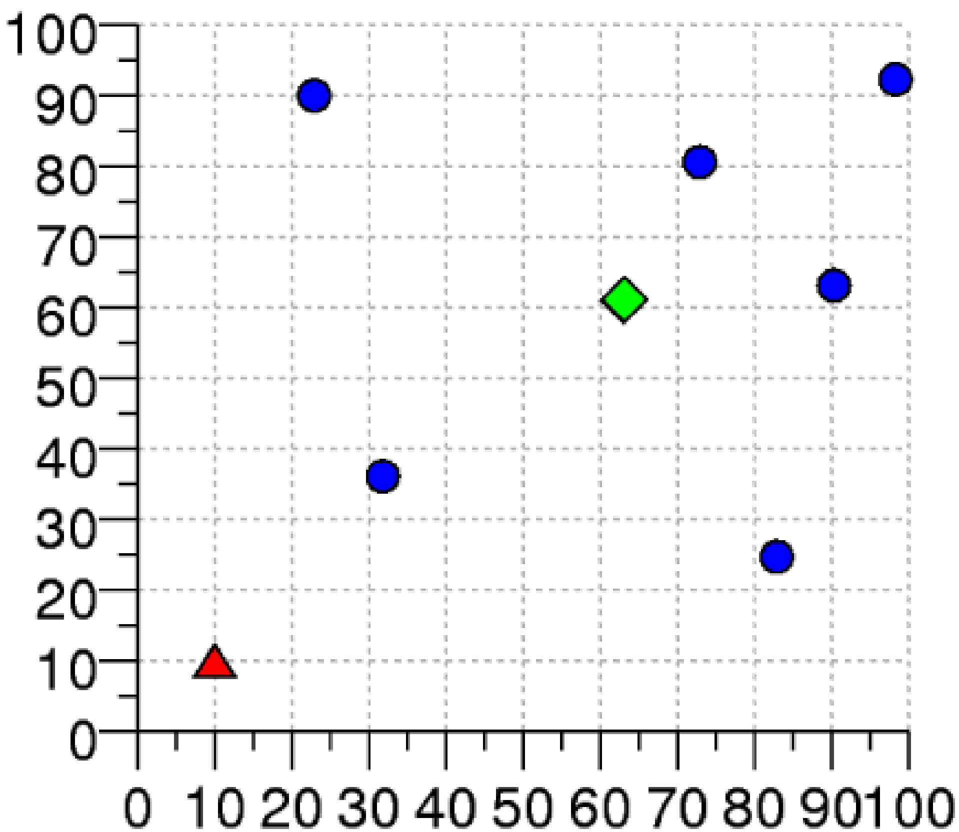
Dougherty et al.

Experimental Round

YOUR GROUP NUMBER: 1 YOUR IDENTIFICATION NUMBER: 1

YOUR IDEAL POINT: (63.1, 61.1)

Each member of your group is assigned an ideal point on a 100 x 100 square as indicated in the first graph below. Your ideal point is indicated by the green diamond. The ideal points of the other members are indicated by the blue circles. The initial alternative (status quo) is indicated by the red triangle.



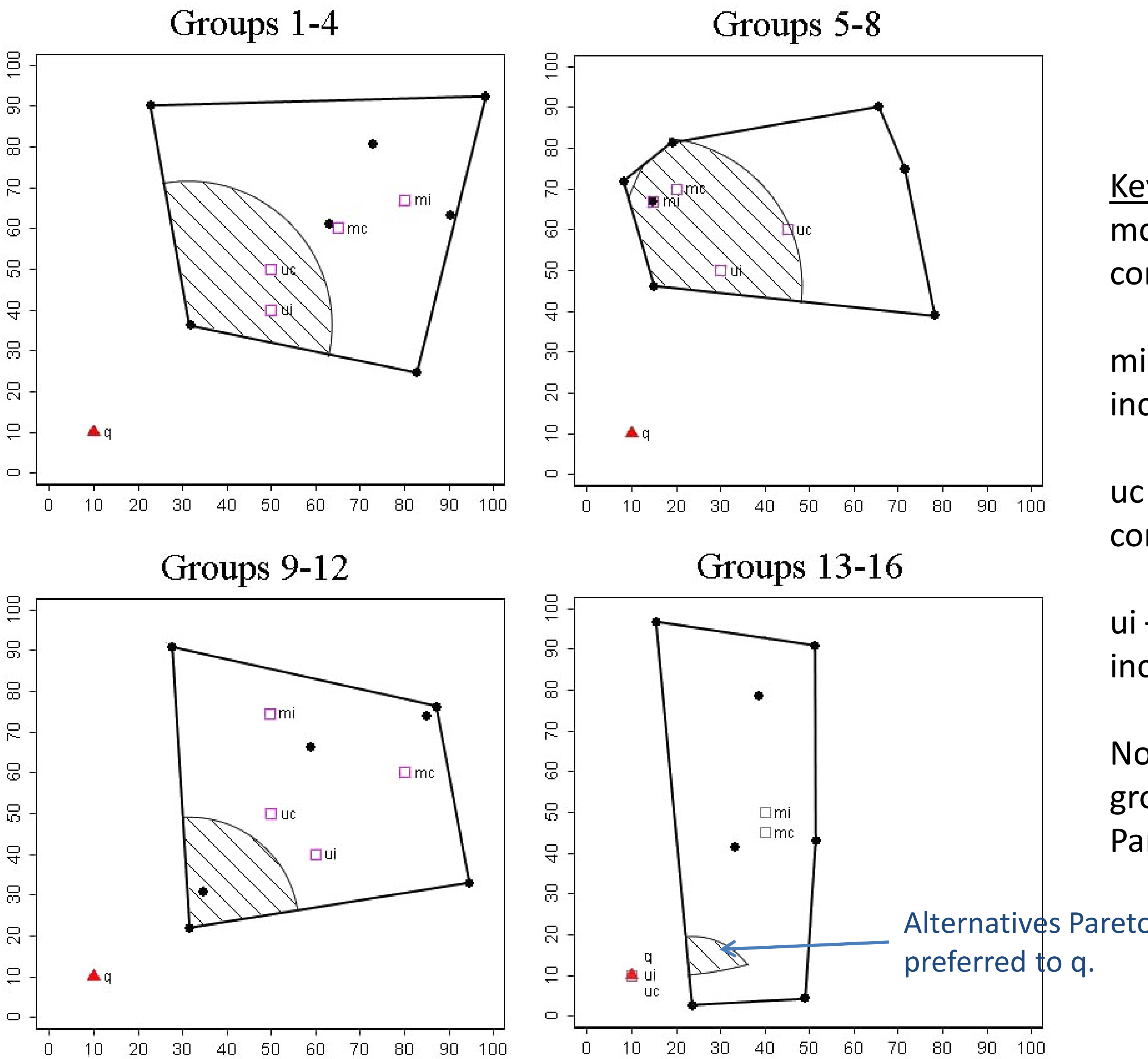
1- No discussion.

2- Subjects are given 30 seconds to consider each proposal.

3- Subjects vote by a show of hands.

4- The process repeats for exactly 10 rounds. Bianco et al. (2007) vote to adjourn.

5- payoffs (based on distance) range between $1 and $15.



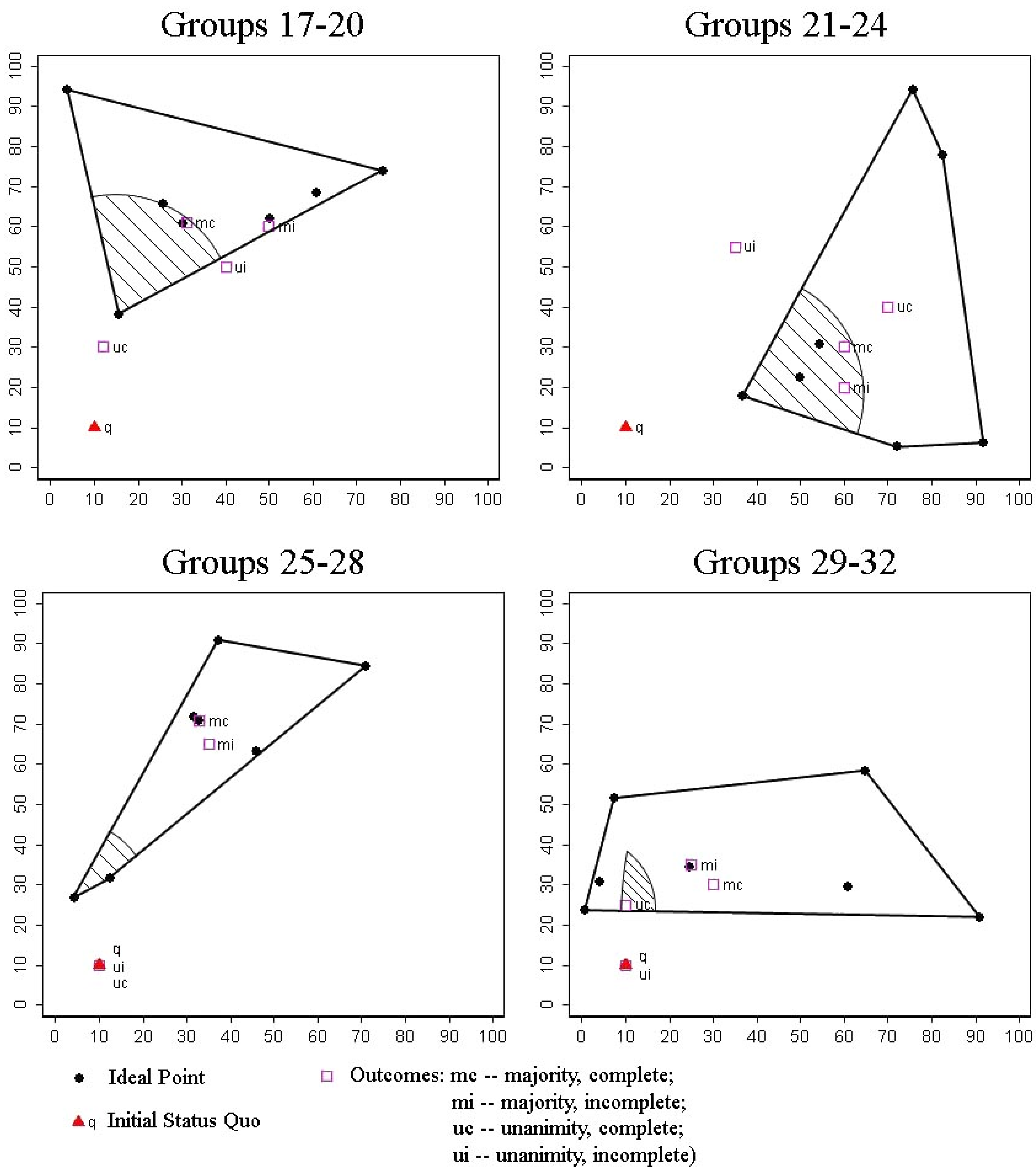
Key:
mc – majority complete info.

mi – majority incomplete info.

uc – unanimity complete info.

ui – unanimity incomplete info.

Note: majority rule groups end in the Pareto set.



100% of the majority rule groups ended in the Pareto set.

But only 43.8% of the majority rule groups ended in the UC set.

<u>Discussion</u>:
Why do these results differ from Bianco et al.?

Experimental Evidence

Table 2 The percentage of final round outcomes in specified regions

	Majority Rule	Unanimity Rule
Nash-Harsanyi Set	31.3%	0.0%
Yolk	68.8%	0.0%
Uncovered Set	43.8%	12.5%
$PP(q_1)\&PO$	31.3%	31.3%
Pareto Set	100.0%	50.0%

Notes: The Nash-Harsanyi set includes any outcome within 5 units of the Nash-Harsanyi solution (always a singleton). Both voting rules are compared in their ability to select an element of the majority rule, uncovered set

Concluding thoughts

- Plott and McKelvey
  - the majority rule core is empty (i.e., no equilibrium).
  - Majority rule creates an *intransitive* order for the entire set of alternatives.
  - Using majority rule, an agenda setter could take us anywhere.
- Dougherty-Edward (2012)
  - Even though we could go anywhere, if we model the proposal process, a rational agenda setter will bring us to an outcome in the Pareto set (i.e., equilibrium again) – regardless of the amendment procedure.
- Uncovered Set
  - Perhaps a stepping stone toward the Banks Set.
- Banks Set
  - If the agenda setter determines the order by which all alternatives are successively voted upon in an elimination procedure, then the power the agenda setter has over outcomes is only as large as the Banks set.