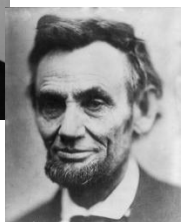


Partial Lecture Notes

# **PLURALITY RULE AND PAIRWISE MAJORITY RULE**

# Plurality Rule and Democratic Decisions



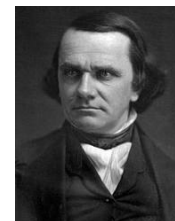
Hypothetical Example: Four Presidential Candidates

B- John Brekenridge,

Bl- John Bell,

D- Stephen Douglas,

L- Abraham Lincoln.



Number of Folks (in millions)

<u>3</u>	<u>6</u>	<u>3</u>	<u>5</u>	<u>2</u>	<u>5</u>	<u>2</u>	<u>4</u>
L	L	B	B	D	D	Bl	Bl
D	Bl	D	Bl	B	Bl	B	D
Bl	D	Bl	D	Bl	B	D	B
B	B	L	L	L	L	L	L

a. Plurality Winner: L with 9 million votes.

We have a winner. But... pick another candidate. What if we run L against him? LINCOLN LOOSES!

# Plurality Rule and Democratic Decisions

<u>3</u>	<u>6</u>	<u>3</u>	<u>5</u>	<u>2</u>	<u>5</u>	<u>2</u>	<u>4</u>
L	L	B	B	D	D	BI	BI
D	BI	D	BI	B	BI	B	D
BI	D	BI	D	BI	B	D	B
B	B	L	L	L	L	L	L

Let's finish this up:

L vs. B -> B wins (9 to 21)

L vs. D -> D wins (9 to 21)

L vs. BI -> BI wins (9 to 21)

Notice, L won under plurality rule, but L was a Condorcet Loser (i.e. he lost to all other candidates in head-to-head contests)!

# Plurality Rule and Democratic Decisions

## B. Problems with plurality rule.

### 3. Plurality can fail to pick a Condorcet Winner.

Condorcet *Winner* Criterion: if a candidate beats all other candidates in pairwise contest, then that candidate should win.

<u>3</u>	<u>6</u>	<u>3</u>	<u>5</u>	<u>2</u>	<u>5</u>	<u>2</u>	<u>4</u>
L	L	B	B	D	D	BI	BI
D	BI	D	BI	B	BI	B	D
BI	D	BI	D	BI	B	D	B
B	B	L	L	L	L	L	L

Notice:

L vs. BI -> BI wins (9 to 21)

B vs BI -> BI wins (10 to 20)

D vs BI -> BI wins (13 to 17)

BI is the Condorcet Winner (the one that beats all others in head-to-head contest), but plurality rule did not select him!

# Plurality Rule and Democratic Decisions

## B. Problems with plurality rule.

4. Plurality ranking rule can contradict the Condorcet ranking.
  - a. Plurality Ranking Rule: everyone votes for their favorite candidate, the candidate with the most votes wins, the candidate with the second most votes comes in second, etc.

<u>3</u>	<u>6</u>	<u>3</u>	<u>5</u>	<u>2</u>	<u>5</u>	<u>2</u>	<u>4</u>
L	L	B	B	D	D	BI	BI
D	BI	D	BI	B	BI	B	D
BI	D	BI	D	BI	B	D	B
B	B	L	L	L	L	L	L

Plurality Ranking Rule:  $L > B > D > BI$ .

So Far:

L vs. BI -> BI wins (9 to 21)

B vs BI -> BI wins (10 to 20)

D vs BI -> BI wins (13 to 17)

L vs. B -> B wins (9 to 21)

L vs. D -> D wins (9 to 21)

Also: B vs. D -> D wins (10 to 20).

Condorcet Ranking:  $BI > D > B > L$ . ... the opposite of the plurality ranking!

# Plurality Rule and Democratic Decisions

## B. Problems with plurality rule.

1. Plurality rule can select against the majority (in a way).
2. Plurality rule can select the Condorcet Loser.
3. Plurality rule can fail to select the Condorcet Winner.
4. Plurality ranking rule can contradict the Condorcet ranking.

... and that's just a start.

# Plurality Rule and Democratic Decisions

## 2. Example: The Agricultural Appropriations Act of 1953

House floor considered how much money to appropriate for agriculture and were stuck on funding the “Soil Conservation Service.”

<u>Proposal</u>	<u>Amount</u>
Committee recommendation (C)	\$250
Anderson (A)	\$200
Whitten (W)	\$225

# Plurality Rule and Democratic Decisions

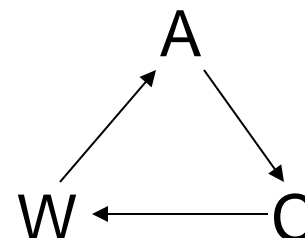
## Coalitions

Committee 1 (108)	Committee 2 (13)	Anderson (126)	Whitten (10)
C	C	A	W
W	A	W	C
A	W	C	A

Problem: vote cycle

Hence, it's not clear what the majority wanted.

The order determined the outcome of the vote!





# Plurality Rule and Democratic Decisions

## D. Other Voting Rules

These examples lead us to ask whether other voting rules are better?