MAY'S THEOREM

Partial Notes

Four Democratic Principles

1. **Decisiveness**
   - The voting rule must specify a unique decision (even if the decision is indifference) for any set of individual preferences.
2. **Anonymity**
   - A voting rule must treat all *voters* alike, in the sense that if any two voters traded ballots, the outcome of the election would remain the same.
   - Ex: if Abdullah Abdullah won with Asa voting for him and Ara voting against, then Abdullah Abdullah should win if Ara voted for him and Asa voted against him.
3. **Neutrality**
   - A voting rule must treat all *candidates* alike, rather than favor one over the other.
   - Ex: if the names Abdullah and Karzai are switched on the ballot but the votes remain the same, then the result should be the same (but favor of the other candidate).
4. **Positive Responsiveness (monotonicity)**
   - if the group decision is indifference or favorable to x, and if individual preferences remain the same except that a single individual changes his/her vote in favor of x, then the group decision should be x (rather than y or remain indifferent).
   - Ex: If Abdullah wins or ties, then he should win if he gains votes without losing votes.

May’s Theorem

- **Theorem: assume a two candidate election with an odd number of voters. Majority rule adheres to these four conditions. Furthermore, these four conditions imply majority rule.**

<u>Proof</u>:

Let the majority rule decision function D be represented by the sum of N(1), N(0), and N(-1), where

N(1) is the number of votes for x,

N(-1) is the number of votes for y, and

N(0) is the number of indifferent voters.

Under simple majority rule (MR)

If D=0, then the social decision is indifference.

If D>0, then the social decision is +1 (for x).

If D<0, then the social decision is -1 (for y).

MR is decisive because it always produces an outcome of +1, 0 or –1.

MR adheres to anonymity because swapping a +1 and -1 among two voters does not affect the sum. Hence the result is unchanged.

May's Theorem

<u>Proof</u>:

That these four conditions imply MR is a little harder to see…

Suppose N(-1)=N(1), then it follows from the first three conditions that D=0. Here's why…

Consider the following seven voters:

(A, A, A, 0, K, K, K).

If A were to win, then swapping all the As and Ks would either produce the reverse result:

f(K, K, K, 0, A, A, A) → K

which violates anonymity because who votes for A and K would determine the result.

Or it would produce the same result:

f(K, K, K, 0, A, A, A) → A

which violates neutrality because A is favored despite N(K)=N(A).

If K were to win, then we have the same problem in reverse.

Hence, N(-1)=N(1) must imply D=0.

May’s Theorem

Proof:

Furthermore, if $N(1)=N(-1)+1$

i.e., (A, A, A, 0, 0, K, K).

then the social decision function must favor A according to positive responsiveness.

Also if $N(1) = N(-1) + (m-1)$ for any $1 < m < N(-1)+1$,

i.e., (A, A, A, 0, A, K, K),

(A, A, A, 0, A, 0, K)….

(A, A, A, A, A, A, A)

then in all these cases the social decision function must favor A according to positive responsiveness.

The voting rule that asserts indifference when $N(1)=N(-1)$ and favors 1 in all the cases mentioned above is simple majority rule.