Rationality & Social Choice

Dougherty, POLS 8000

Social Choice



- A. Background
 - 1. Social Choice examines how to aggregate individual preferences *fairly*.
 - a. Voting is an example.
 - b. Think of yourself writing a constitution for a new country, like Egypt after the Arab Spring. You have to engineer a voting system that promotes certain democratic principles that are consistent.

B. Individual Rationality

- 1. Components:
 - a. Alternatives something one can choose.
 - b. Preferences a liking of one thing compared to another.
 - 1) Preference relations
 - a) Weak Preference Relation

For two alternatives x and y, "x R y" means "x is weakly preferred to y" (or "x is at least as preferred as y").

b) Strict Preference Relation

For two alternatives x and y, "x P y" means "x is strictly preferred to y" (or "x is preferred to y").

c) Indifference

For two alternatives x and y, "x I y" means an individual is indifferent between "x and y."



Elements

2) Other Depictions

a) utility – higher numbers imply more preferred alternatives.

Ex: $u_i(w) = 2$, $u_i(x) = 8$, $u_i(y) = 6$, $u_i(z) = 6$.

b) preference lists -- alternatives higher on the list are more preferred.



Elements

- 2. Three properties of preferences
 - a. Reflexive: $\forall x \in X, x R_i x$.

A preference relation is reflexive if and only if it is weakly preferred to itself.

b. Complete: $\forall x, y \in X$, s.t. $x \neq y$, $xR_iy \lor yR_ix$. A preference relation is complete if and only if for any two alternatives x and y *either* xR_iy *is true or* yR_ix *is* true" (completeness rules out the case where we cannot compare).

c. Transitive: $\forall x, y, z \in X, xR_i y \& yR_i z \rightarrow xR_i z$.

A preference relation is transitive if and only if for any three alternatives x, y, and z, if x is weakly preferred to y and y is weakly preferred to z, then x is weakly preferred to z.





Elements

3. Rationality

- a. If properties a-c are true there will always be a set of alternatives that an individual prefers at least as much as all other alternatives.
- b. If properties a-c are true, and individuals choose according to their preferences, then they will choose something from their most preferred set.
 - 1) This would make them rational under some definitions.

Discussion

- 1. Are people rational?
- 2. Does this mean they make reasonable choices?
- 3. Does this mean they are self-interested?

<u>Note</u>: the dictionary definition of rationality and the economic definition of rationality are not the same!

A. Preference aggregation rule: a function that aggregates individual preference rankings into a complete and reflexive social ranking.

- B. Three Preference aggregation rules
 - 1. Plurality Rule -- the candidate with the most votes wins (that is, the most first placed votes if everyone votes sincerely).

Ex:	<u>5 voters</u>	<u>3 voters</u>	<u>4 voters</u>
	А	В	С
	С	С	A
	В	A	В

A wins. Furthermore, A > C > B.

- Used in Canada, India, Iran, Mexico, South Korea, Thailand, the United Kingdom, and the United States to name a few.
- Among people living in a democracy, most of the world's people live under plurality rule, but more democratic countries use proportional representation.

2. Majority Rule with a Runoff (MRR) - the candidate that receives a majority in the first round wins. If no candidate wins a majority, the two candidates with the most votes go to the second round and the candidate who receives a majority wins.

Ex:	<u>5 voters</u>	<u>3 voters</u>	<u>4 voters</u>
	А	В	С
	С	С	А
	В	А	В

- No candidate receives a majority because no candidate has 7 or more votes.
- Candidates A and C go to round 2 because they have the most votes.
- Candidate C beats candidate A (7 votes to 5).
 C wins. Furthermore, A > C > B.

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 Used in legislative elections in France; presidential elections in Austria, Bulgaria, Chile, France, Portugal, and Ukraine. It is also used in U.S. local elections in Georgia, Louisiana, and parts of Florida.

3. Borda Count - Each voter ranks the alternatives according to their preferences giving greater numbers to their most preferred alternatives. These numbers are then added and the alternative with the largest total wins.

Ex:	<u>5 voters</u>	<u>3 voters</u>	<u>4 voters</u>
	A (3)	B (3)	C (3)
	C (2)	C (2)	A (2)
	B (1)	A (1)	B (1)

Borda Count

A: 3(5) + 1(3) + 2(4) = 26B: 1(5) + 3(3) + 1(4) = 18C: 2(5) + 2(3) + 3(4) = 28

> C wins because it has the largest count. Furthermore, C > A > B.

- 3. Borda Count
 - Used in Slovenia to elect the member of its National Assembly who represents the ethnic Italians and the member who represents the ethnic Hungarians. It is also used to nominate presidential candidates in Kiribati (Reilly, 2002), to determine the Most Valuable Player in Major League Baseball, and to nominate Heisman Trophy winners.

C. 1984 NY Democratic Primary

Jackson coalition	25%	JHM
Hart coalition	35%	ΗJΜ
Mondale coalition	40%	MJH

Practice: who wins under plurality rule, MRR, and Borda Count?

(note: preferences are left to right, not top to bottom)







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1) Plurality:







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- 1) Plurality:
 - 1) Mondale Wins (M − 40%, H − 35%, J − 25%).







Jackson coalition	25%	JHM
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- 1) Plurality:
 - 1) Mondale Wins.
- 2) Majority with Runoff:
 - 1) no one has a majority in the first round so Hart and Mondale go to the second round.





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- 1) Plurality:
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- 2) Majority with Runoff:
 - 1) no one has a majority in the first round so Hart and Mondale go to the second round.
 - 2) Hart wins by majority in the second round (60% to 40%). Hart wins.



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3) Borda Count:

Jackson: 3(.25) + 2(.35) + 2(.40) = 2.25

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Jackson:3(.25) + 2(.35) + 2(.40) = 2.25Hart:3(.35) + 2(.25) + 1(.40) = 1.95

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Hart:	3(.35) + 2(.25) + 1(.40) = 1.95
Mondale:	3(.40) + 1(.35) + 1(.25) = 1.8

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Jackson wins.

D. Discussion

- Notice that different candidates won depending upon which voting rule we used.
- When we talk about popular will, which voting rule are we referring to?
- If you observe that a country elects say Jones, can you say that the choice of Jones represents the interest of the people?

- Note: This should not suggest that different voting rules will always select different candidates, but it should suggest that they can (and by the way often do) select different candidates.
- Since we know that the voting rule matters, the natural question is which voting rule is best.

Trump, Condorcet and Borda

- Kurrild-Klitgaard (2017) shows that there may have been a vote cycle among Republican candidates in the last Presidential Election (i.e., intransitive social preferences).
 - a. <u>Vote Cycle</u>: In an April 2105 poll (before Trump entered the race), a majority of Republicans surveyed preferred Walker to Bush, another majority preferred Bush to Cruz, and a third majority preferred Cruz to Walker -- violating transitivity.
 - b. How this can happen
 - $\begin{array}{cccc}
 \frac{1 & 2 & 3}{X & Z & Y} \\
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 7 & Y & X
 \end{array}$ $\begin{array}{cccc}
 X & vs & Z: Z & wins (2 to 1) \\
 Z & vs & Y: Y & wins (2 to 1) \\
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 \end{array}$

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 - 1
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Note, pairwise majority rule can violate transitivity. Plurality, MRR, and Borda cannot.



- A. Background.
 - 1. Arrows Impossibility Theorem may be the single most important theorem in the social sciences.
 - a. In the early part of the 20th century, philosophers attempted to link the liberal tradition (in England) with the communitarian tradition (in Continental Europe) via utilitarianism.
 - 1) Arrow's theorem ended that.
 - b. At the same time, economists were working on a way to choose policies among all Pareto efficient policies with a social welfare function.
 - 1) Arrow's theorem ended that.
 - c. Quotes:

"It is not stating the case too strongly to say that Arrow's theorem and the research that it inspired wholly undermine the general applicability or meaning of concepts such as the 'public interest' and 'community goals'" (Peter Ordeshook).

"The search of the great minds of recorded history for the perfect democracy, it turns out, is the search for a chimera, for a logical contradiction. Now scholars all over the world – in mathematics, politics, philosophy and economics – are trying to salvage what can be salvaged from Arrow's devastating discovery that is to mathematical politics what Kurt Gödel's 1931 impossibility-of-proving-consistency theorem is to mathematical logic" (Paul Samuelson, Nobel Prize 1970).



For at least three alternatives and at least two voters, no preference aggregation rule adheres to five fairness conditions (Austen-Smith and Banks, 1999, version).

•*Preference Aggregation Rule* (PAR) – takes a preference profile, ρ , as input and generates a binary preference relation for society that is reflexive and complete.



Conditions:

- **1. Unrestricted Domain (U)** The domain of the PAR must include all possible combinations of individual preference orderings.
- 2. Transitivity (T) In the social ranking.
- **3. Pareto (P)** If everyone strictly prefers *a* to *b*, then society must strictly prefer *a* to *b*.
- **4.** Independence of Irrelevant Alternatives (IIA) for two different profiles ρ and ρ' : if for all $i \ a \succeq_i b$ if and only if $a \succeq_i b$, then $a \succeq b$ if and only if $a \succeq b$. In other words, for all pairs of alternatives a and b the social ranking between a and b depends only on individual preference rankings between a and b.
- 5. Non-dictatorship (N-D) there is no individual whose preferences determine the social ranking of all alternatives, regardless of how other individuals rank the alternatives.
- Stated differently: A PAR that satisfies U,T,P and IIA for three or more alternatives must be a dictatorship.

<u>Arrow's Theorem</u>

- C. Intuition behind the theorem
 - 1. Ramsey Center has some money to buy new equipment.
 - a. Students propose following expenditures:
 - Elliptical machines (e), camping gear (c), squat cages (s), more rock climbing (r).

Center wants to rank spending priorities based on student preferences.



Unrestricted Domain (U) – says that any order of individual preferences are allowed. Here's one for five individuals.

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Individual Preferences	Social Preferences	Ρ
<u>1 2 3 4 5</u> c e e c s	е	a th
sscse —	→ S	Х
rrser — ecrrc	C	
	r	

Pareto (P) –if everyone in a society prefers x to y, then society should prefer x to y.

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Individual Preferences	Social Preferences	IIA – for any pair of alternatives
<u>1 2 3 4 5</u> c e e c s	е	(such as e and s), the social ranking of that pair of alternatives
sscse —	→ S	(e and s)
rrser —		
ecrrc	6	
	r	

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<u>12345</u>	e ran
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IA – for any pair of alternatives such as e and s), the social anking of that pair of alternatives e and s) should be independent of the individual rankings of other pairs (such as c and r).

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 - 1. Ramsey Center has some money to buy new equipment.
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Hence, if one or more individuals switched r and c (but left their ranking of e and s unchanged), the social ranking of e and s should be unchanged.

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Hence, if one or more individuals switched r and c (but left their ranking of e and s unchanged), the social ranking of e and s should be unchanged. This is true for any "exalted" pair (e and s in this case) and any number of switches of "irrelevant pairs" (c & r, c & e, c & s, etc...). Individual Preferences Social Preferences IIA – for any pair of alternatives

<u>12345</u>	0
ceecs	e
ss <mark>c</mark> se	→ S
r r s e c	
ecrrr	C C
	r

IIA – for any pair of alternatives (such as e and s), the social ranking of that pair of alternatives (e and s) should be independent of the individual rankings of other pairs (such as c and r).

- D. Pedagogical Proof
 - see other power point.

- E. Conditions Voting Rules Violate
 - 1. Drunkered if the town drunk prefers x to y, society should prefer y to x. If he is indifferent between x and y, then so should be society.
 - a. Is this voting rule consistent, in the sense of making a well defined choice?
 - 1) Yes.
 - b. Which of Arrow's conditions does it violate?
 - 1) Pareto.
 - c. <u>Point</u>: there are plenty of voting rules that are consistent. We need a voting rule that is consistent but also reasonable. Arrow's theorem is about the problem of requiring too many conditions of reasonableness.
 - 1) Since reasonableness is a matter of opinion, it is useful to impose only mild conditions like U, T, P, BI, and D.

- F. Discussion
 - 1. What do you think about Arrow's theorem?
 - 2. Is it a serious problem for determining the public interest?
 - 3. Would we alleviate the problem if assumed individuals were civically minded?
 - 4. It appears that the only way out of Arrow's theorem is to relax one of the conditions. In which case, which condition is least important?