NORMAL FORM (SIMULTANEOUS MOVE) GAMES

For These Games

- Choices are simultaneous – made *independently* and without observing the other players' actions

- Players have *complete information*, which means they know the structure of the game, actions, and preferences (both their own and the other players).

- Such information is *common knowledge* (all players know that all players know this)

Prisoners' Dilemma

Player 2 (Column)

Player 1 (Row)

	C	D
C	3 , 3	0 , 5
D	5 , 0	2 , 2

How would you play?
I need a volunteer.

Analysis

Player 2 (Column)

Player 1 (Row)

	C	D
C	3 , •	0 , •
D	5 , •	2 , •

How a Rational Actor Would Play:

For each of Column’s actions, what is Row’s rational choice?

Analysis

		Player 2 (Column) C	Player 2 (Column) D
Player 1 (Row)	C	3 , •	0 , •
	D	5 , •	2 , •

How a Rational Actor Would Play:

For each of Column's actions, what is Row's rational choice?

Note:

5 > 3 and 2 > 0. So folks have argue that D is better than C no matter what the other player chooses.

Same for player 2.

Summary: Prisoner's Dilemma

		Column C	Column D
Row	C	3 , 3	0 , 5
	D	5 , 0	2 , 2

- D is a **dominant strategy** for each player (it is best regardless of what the other player chooses)
- Individually rational behavior predicts the outcome (D, D).
- Yet (C, C) is Pareto superior to (D, D)
  - In a sense, (D, D) might be considered individually rational but collectively irrational.

Strict dominance

For player 1, strategy s *strictly dominates* strategy t if

$$u_1(s,a_{-i}) > u_1(t,a_{-i}) \text{ for all } a_{-i}$$

… where a_{-i} is all the actions of the other player

	w	x	y	z
s	2, ·	4, ·	0, ·	-1, ·
t	0, ·	2, ·	-2, ·	-4, ·

Equivalently, strategy t *is strictly dominated by* strategy s.

Strict dominance

Strategy s is a *strictly dominant* strategy for player 1 if

$u_1(s,a_{-i}) > u_1(t,a_{-i})$, $u_1(s,a_{-i}) > u_1(r,a_{-i})$, ...

for all of 1's possible actions, t, r, etc. and for all a_{-i}

	w	x	y	z
s	2, ·	4, ·	0, ·	-1, ·
t	0, ·	2, ·	-2, ·	-4, ·
r	1, ·	3, ·	-1, ·	-2, ·

Equivalently, s is strictly dominant if it strictly dominates all other strategies

Strict dominance

- Strictly dominant strategy $\Rightarrow$ always a best response
- Strictly dominated strategy $\Rightarrow$ never a best response
- Alternative solution concept: iterated dominance
  - **Iterated elimination of strictly dominated strategies**
  - I will refer to the outcomes that remain after an iterated elimination of strictly dominated strategies as a strictly dominant strategy equilibria (SDSE).
  - Game is "dominance solvable" if the SDSE is unique
  - Nash equilibria, coming soon, are a subset of action profiles that "survive" iterated elimination (i.e. $NE \subseteq SDSE$).

Example: Iterated elimination of dominated strategies

		Column		
		Left	**Center**	**Right**
	Up	1, 1	0, 1	2, 4
Row	Middle	2, 2	1, 3	4, 1
	Down	3, 3	2, 1	1, 2

<u>Rule of thumb</u>: When eliminating rows, look only at row's payoffs. When eliminating columns, look only at column's payoffs.

Example: Iterated elimination of dominated strategies

		Column		
		Left	Center	Right
Row	Up	1, 1	0, 1	2, 4
	Middle	2, 2	1, 3	4, 1
	Down	3, 3	2, 1	1, 2

Example: Iterated elimination of dominated strategies

		Column		
		Left	Center	Right
Row	Up	**1**, 1	**0**, 1	**2**, 4
	Middle	**2**, 2	**1**, 3	**4**, 1
	Down	3, 3	2, 1	1, 2

Middle dominates up, because **2>1**, **1>0**, and **4>2**.
So we can eliminate up.

Example: Iterated elimination of dominated strategies

Column

Row		Left	Center	Right
	Up	~~1, 1~~	~~0, 1~~	~~2, 4~~
Row	Middle	2, 2	1, 3	4, 1
	Down	3, 3	2, 1	1, 2

Now try to eliminate columns.

Example: Iterated elimination of dominated strategies

		Column: Left	Center	Right
Row	Up	~~1, 1~~	~~0, 1~~	~~2, 4~~
	Middle	2, 2	1, 3	4, 1
	Down	3, 3	2, 1	1, 2

Now try to eliminate columns.

Example: Iterated elimination of dominated strategies

		Column		
		Left	Center	Right
	Up	~~1, 1~~	~~0, 1~~	~~2, 4~~
Row	Middle	2, **2**	1, 3	4, **1**
	Down	3, **3**	2, 1	1, **2**

Now try to eliminate columns.

Left dominates right because **2>1** and **3>2**.

Example: Iterated elimination of dominated strategies

		Column: Left	Center	Right
Row	Up	~~1, 1~~	~~0, 1~~	~~2, 4~~
	Middle	2, **2**	1, 3	4, **1**
	Down	3, **3**	2, 1	1, **2**

Now try to eliminate columns.

Left dominates right because **2>1** and **3>2**.

Example: Iterated dominance

		Column		
		Left	Center	Right
	Up	~~1, 1~~	~~0, 1~~	~~2, 4~~
Row	Middle	2, 2	1, 3	4, 1
	Down	3, 3	2, 1	1, 2

Now try to eliminate rows again.

Example: Iterated elimination of dominated strategies

		Column		
		Left	Center	Right
	Up	~~1, 1~~	~~0, 1~~	~~2, 4~~
Row	Middle	~~**2**, 2~~	~~**1**, 3~~	4, 1
	Down	**3**, 3	**2**, 1	1, 2

Now try to eliminate rows again.

Down dominates middle because **3>2** and **2>1**.

Example: Iterated elimination of dominated strategies

		Column		
		Left	Center	Right
	Up	~~1, 1~~	~~0, 1~~	~~2, 4~~
Row	Middle	~~2, 2~~	~~1, 3~~	4, 1
	Down	3, **3**	2, **1**	1, 2

Now try to eliminate columns again.

Left dominates center because **3>1**.

Example: Iterated elimination of dominated strategies

		Column		
		Left	Center	Right
Row	Up	~~1, 1~~	~~0, 1~~	~~2, 4~~
	Middle	~~2, 2~~	~~1, 3~~	4, 1
	Down	3, 3	2, 1	1, 2

SDSE = {Down, Left}

…because a single outcome remains, we call the game “dominance solvable.”

Practice: Iterated elimination of dominated strategies

Column

Row		x	y	z
	A	2, 3	-16, 2	5, 0
Row	B	5, 6	4, 6	6, 4
	C	8, 0	3, 10	1, 8

Nash equilibrium

Informally, a Nash equilibrium is an action profile such that *no player has a unilateral incentive to "deviate"* (holding all other players' choices constant, each player's choice is rational)

Nash Equilibrium: PD

Column

Row		C	D
	C	3, 3	0, 5
	D	5, 0	1, 1

How do I find Nash equilibria?

Determine the best responses, that is the best action (or strategy) for a player given the actions (or strategies) played by opponents. The best responses for each player intersect at the Nash equilibrium.

Nash Equilibrium: PD

Column

		C	D
Row	C	3, 3	0, 5
	D	5, 0	1, 1

Given column plays C, what is best response for Row?

Nash Equilibrium: PD

Column

Row		C	D
	C	3, 3	0, 5
	D	5, 0	1, 1

Given column plays C, what is best response for Row?

D because 5 > 3.

Nash Equilibrium: PD

Column ↓

Row		C	D
	C	3, 3	0, 5
	D	(5), 0	1, 1

Given column plays C, what is best response for Row?

D because 5 > 3.

Let’s circle 5 because it indicates one of the best responses.

Nash Equilibrium: PD

Column ↓

Row

	C	D
C	3, 3	0, 5
D	(5), 0	1, 1

Given column plays D, what is best response for Row?

Nash Equilibrium: PD

Column

		C	D
Row	C	3, 3	0, 5
	D	5, 0	1, 1

Given column plays D, what is best response for Row?

D because 1 > 0.

Nash Equilibrium: PD

Column ↓

Row		C	D
	C	3, 3	0, 5
	D	(5), 0	(1), 1

Given column plays D, what is best response for Row?

D because 1 > 0.

Let's circle 1 because it indicates one of the best responses.

Nash Equilibrium: PD

Column

Row

	C	D
C	3, 3	0, 5
D	5, 0	1, 1

Given “Row” plays C, what is best response for Column?

Nash Equilibrium: PD

Column

Row

	C	D
C	3, 3	0, 5
D	5, 0	1, 1

Given “Row” plays C, what is best response for Column?

D because 5 > 3.

Nash Equilibrium: PD

Column

Row →

	C	D
C	3, 3	0, (5)
D	(5), 0	(1), 1

Given "Row" plays C, what is best response for Column?

D because 5 > 3.

Let's circle 5 because it indicates one of the best responses.

Nash Equilibrium: PD

Column

Row

	C	D
C	3, 3	0, 5
D	5, 0	1, 1

Given "Row" plays D, what is best response for Column?

Nash Equilibrium: PD

Column

Row →

	C	D
C	3, 3	0, (5)
D	(5), 0	(1), 1

Given "Row" plays D, what is best response for Column?

D because 1 > 0.

Nash Equilibrium: PD

Column

Row		C	D
	C	3, 3	0, 5
→	D	5, 0	1, 1

Given “Row” plays D, what is best response for Column?

D because 1 > 0.

Let’s circle 1 because it indicates one of the best responses.

Nash Equilibrium: PD

Column

Row →

	C	D
C	3, 3	0, (5)
D	(5), 0	(1), (1)

Where the best responses intersect {D,D} is a Nash Equilbrium.

N.E. = {D,D}

Note: equilibria are always stated in terms of strategies (or actions), never in terms of payoffs in the outcomes.

Remarks

- An action profile is not a Nash equilibrium when at least one player has an incentive to *unilaterally* deviate (because the criterion is individual rationality, joint deviations are irrelevant)

- An "incentive to deviate" means that utility from another action must be *strictly* better than in the candidate action profile (indifferent between two different actions giving the highest utility)

- A Nash equilibrium is a *"stable" outcome* in the sense that it is *self-enforcing*

Practice: Nash Equilibrium

		Column x	Column y	Column z
Row	A	2, 3	-16, 2	5, 0
Row	B	5, 6	4, 6	6, 4
Row	C	8, 0	3, 10	1, 8

Stag hunt

The story

- Two hunters
- Capturing a stag requires joint effort
- A hare can be captured with individual effort
- Hunting the stag and a hare are mutually exclusive
- The stag is more valuable than the hare, which is still better than nothing

Column

Row

How would we fill in actions and payoffs in matrix?

Stag hunt

The story

- Two hunters
- Capturing a stag requires joint effort
- A hare can be captured with individual effort
- Hunting the stag and a hare are mutually exclusive
- The stag is more valuable than the hare, which is still better than nothing

Column

What is SDSE?

Row

What is NE?

Battle of the Sexes

The story

- Husband and wife
- Choice of two activities: Ballet, Sports
- Wife prefers sports, husband prefers ballet
- Both prefer being together to being apart – each gets zero if they are apart.

Husband

Wife

How would we fill in actions and payoffs in matrix?

Battle of the Sexes

The story

- Husband and wife
- Choice of two activities: Ballet, Sports
- Wife prefers sports, husband prefers ballet
- Both prefer being together to being apart – each gets zero if they are apart.

Husband

Wife

What is NE?

What is SDSE

Matching Pennies

The game

- Each player chooses Heads or Tails
- Row wins if choices match, looses if they differ.
- Column wins if choices differ, looses if they match.

Column

Row

How would we fill in actions and payoffs in matrix?

Matching Pennies

The game

- Each player chooses Heads or Tails
- Row wins if choices match, looses if they differ.
- Column wins if choices differ, looses if they match.

Column

Row

What is NE?

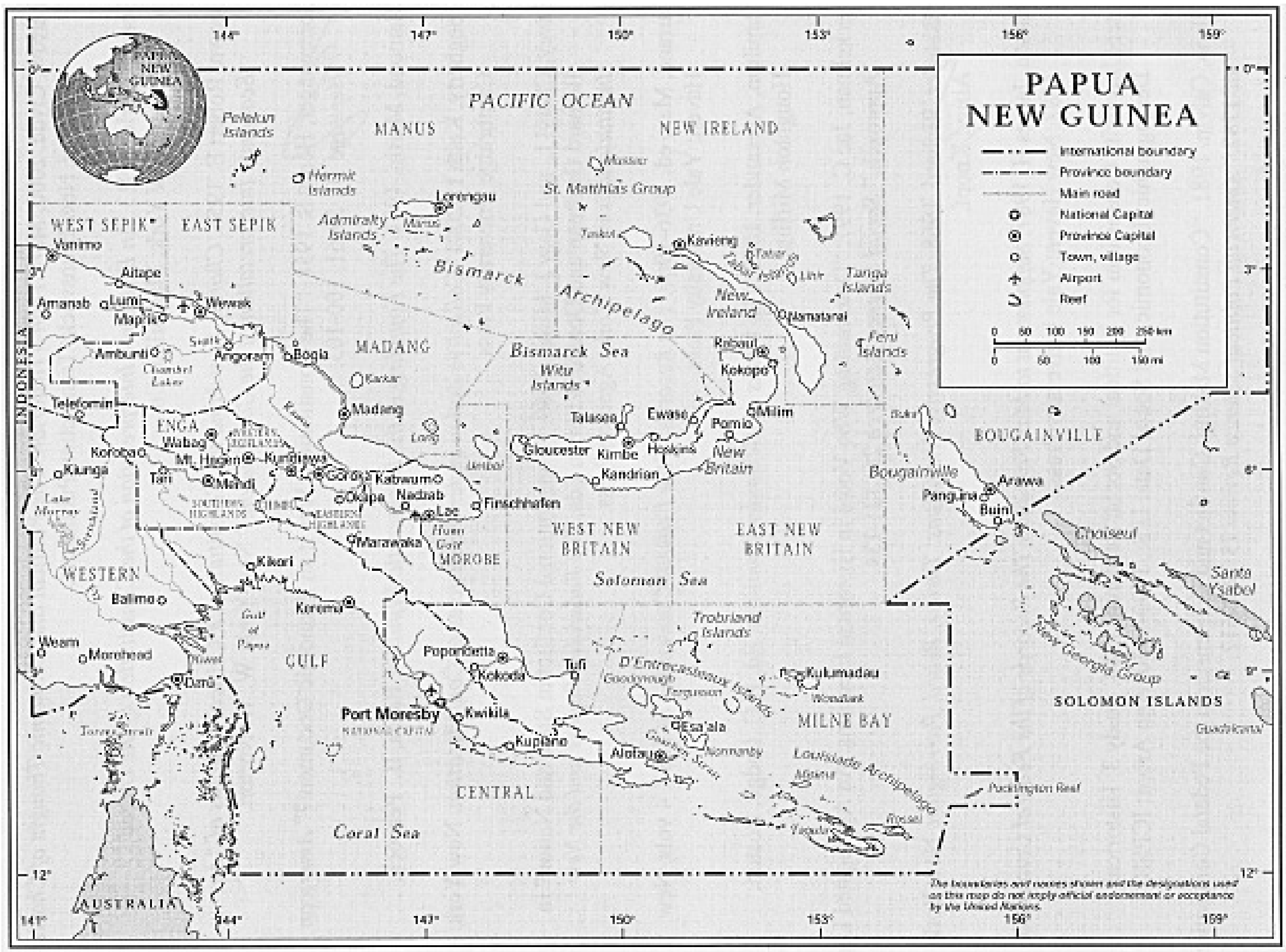
What is SDSE?

Application: Battle of the Bismarck Sea

- This is a zero-sum game, which we will analyze using the concepts we already know.

Application: Battle of the Bismarck Sea

- The story



Application: Battle of the Bismarck Sea

- Normal Form Game

Kimura

Kenney

	North	South
North	2	2
South	1	3

Kimura

Kenney

	North	South
North	2, -2	2, -2
South	1, -1	3, -3

What is NE?

What is SDSE?

Application: Battle of the Bismarck Sea

- Historical Outcome:
  - Keeney searched north.
  - Kimura sailed north.
  - Allies bombed the convoy for three days.
    - Of the 7,000 Japanese troops, only 800 reached Lae.
    - Only 13 allies were killed with the loss of 6 small planes.

Interpretation of multiple equilibria

- All Nash equilibria are "stable" and consistent with individually rational behavior.

- Nash equilibrium does not predict *which* equilibrium will be the outcome nor does it explain *how* players' actions settle on action profiles.

- It only tells us that once players actions settle on a Nash equilibrium profile, they have no incentive to change their behavior.

Summary

- Strategic games
  - Players
  - Actions for each player
  - Preferences over action profiles

- Nash equilibrium
  - Action profile such that no player has a unilateral incentive to deviate
  - Predicts stable outcomes, but may not be unique

- Key skills to develop
  - Translating verbal theories or models into strategic games
  - Determine whether an action profile is a SDSE
  - Determine whether an action profile is a Nash equilibrium