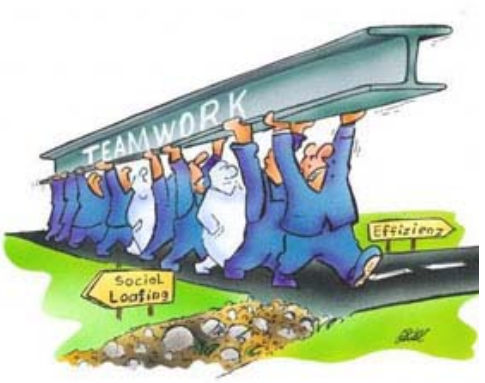
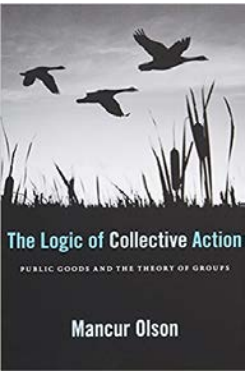
N-PLAYER GAMES

The Logic of Collective Action



Accepted wisdom in the 1960s:

- If everyone in a group has an interest in common, then they will act collectively to achieve that interest.
- The *Logic of Collective Action* challenged that wisdom by noting that individuals have an incentive to free-ride on collective efforts.



The Logic of Collective Action

- Free-Rider Problem (collective action problem)
  - Occurs when individuals consume a good that gives them benefit regardless of whether they pay the cost.
    - *Note*: "good" can include services and other things.
    - *Key ingredient*: individuals cannot be excluded from consuming the good.
    - Examples
      - Voluntary contribution to national defense.
      - Electing your favorite candidate.
      - protest movement.

N-Player Game

- All of the n-player games we study today will be collective action problems of different sorts.

- To make our study of these games easier, assume …
  - 10 players,
  - Each player contributes or defects (i.e. they don't contribute variable amounts),
  - payoffs are symmetric (i.e. the game is identical for all players).

- In n-player normal form games, row plays against the contributions of everyone else combined. Hence, each game contains row's payoff only. We then presume that everyone behaves the same because the payoffs are symmetric.

Prisoner's Dilemma

- Ex: The extra credit game.

  - Benefits

    (#contributors * 3 * 2) / 10

  - Costs

    3 for contributing and 0 for defecting.

  - Payoff

    benefits – costs.

Prisoner's Dilemma

- Ex: The extra credit game.

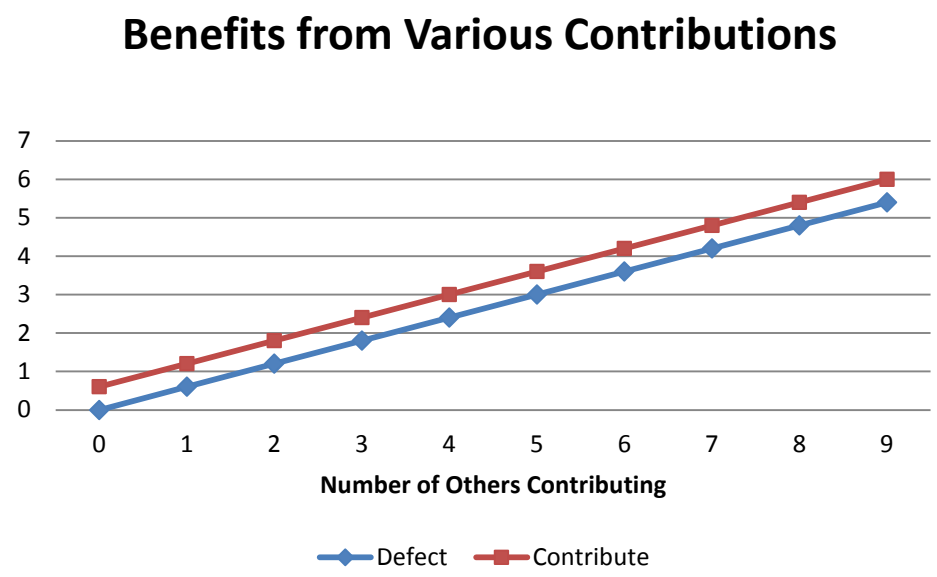
		A		B		
Number of others		**Benefits**			**Payoff**	
contributing		**Defect**	**Contribute**		**Defect**	**Contribute**
0		(2*0)/10	(2*3)/10		A-0	B-3
1		(2*3)/10	(2*6)/10		A-0	B-3
2		(2*6)/10	(2*9)/10		A-0	B-3
3		(2*9)/10	(2*12)/10		A-0	B-3
4		(2*12)/10	(2*15)/10		A-0	B-3
5		(2*15)/10	(2*18)/10		A-0	B-3
6		(2*18)/10	(2*21)/10		A-0	B-3
7		(2*21)/10	(2*24)/10		A-0	B-3
8		(2*24)/10	(2*27)/10		A-0	B-3
9		(2*27)/10	(2*30)/10		A-0	B-3

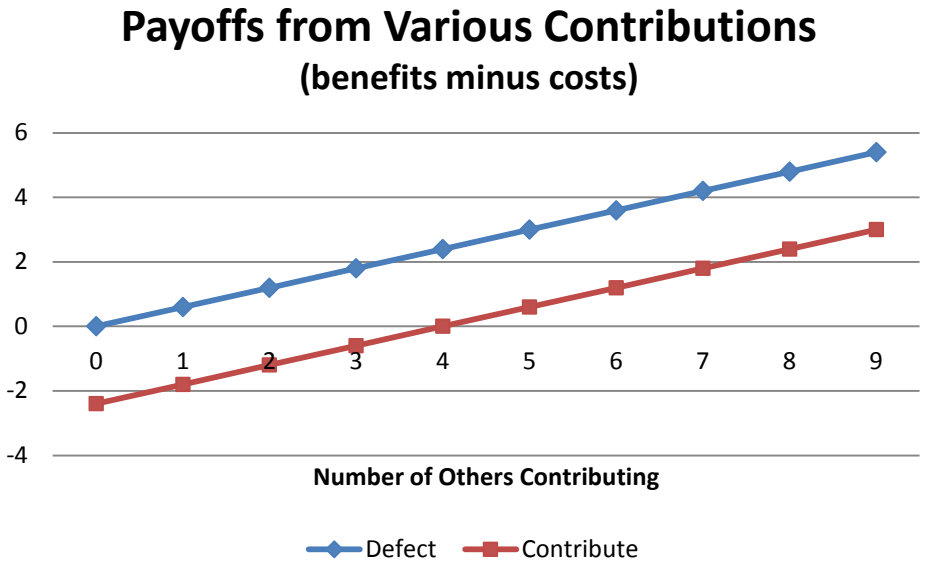
Prisoner's Dilemma

- Ex: The extra credit game.

Number of others		Benefits			Payoff	
contributing		Defect	Contribute		Defect	Contribute
0		0	0.6		0	-2.4
1		0.6	1.2		0.6	-1.8
2		1.2	1.8		1.2	-1.2
3		1.8	2.4		1.8	-0.6
4		2.4	3		2.4	0
5		3	3.6		3	0.6
6		3.6	4.2		3.6	1.2
7		4.2	4.8		4.2	1.8
8		4.8	5.4		4.8	2.4
9		5.4	6		5.4	3

Prisoner's Dilemma





Prisoner's Dilemma: normal form

Number of Others Contributing

Row		9	…	4	3	2	1	0
	contribute	(2*30)/10-3		(2*15)/10-3	(2*12)/10-3	(2*9)/10-3	(2*6)/10-3	(2*3)/10-3
	defect	(2*27)/10		(2*12)/10	(2*9)/10	(2*6)/10	(2*3)/10	(2*0)/10

Row		9	…	4	3	2	1	0
	contribute	3		0	-0.6	-1.2	-1.8	-2.4
	defect	5.4		2.4	1.8	1.2	0.6	0

Prisoner's Dilemma: normal form

Number of Others Contributing

Row	9	…	4	3	2	1	0
contribute	(2*30)/10-3		(2*15)/10-3	(2*12)/10-3	(2*9)/10-3	(2*6)/10-3	(2*3)/10-3
defect	(2*27)/10		(2*12)/10	(2*9)/10	(2*6)/10	(2*3)/10	(2*0)/10

Row	9	…	4	3	2	1	0
contribute	3		0	-0.6	-1.2	-1.8	-2.4
defect	(5.4)		(2.4)	(1.8)	(1.2)	(0.6)	(0)

- Determine best responses for Row.
- Regardless of the amount contributed by others, in this game it is always better to defect.
- Yet everyone would be better off if they contributed (because 3 > 0).

Assurance Game

- Ex: Drain a Meadow.

  “Two neighbors may agree to drain a meadow, which they possess in common; because ‘tis easy for them to know each others mind; and each must perceive, that the immediate consequence of his failing in his part, is, the abandoning the whole project. But ‘tis very difficult, and indeed impossible, that a thousand persons shou’d agree in any such action; it being difficult for them to concert so complicated a design, and still more difficult for them to execute it; while each seeks a pretext to free himself of the trouble and expence, and wou’d lay the whole burden on others.”

  -David Hume

Assurance Game

- Ex: Ten people drain a meadow. For each additional contributor, the chances of being successful increases at an increasing rate.

  - Benefits

    #contributors^2

  - Costs

    6 for contributing and 0 for defecting.

  - Payoff

    benefits – costs.

Assurance Game

- Drain a Meadow.

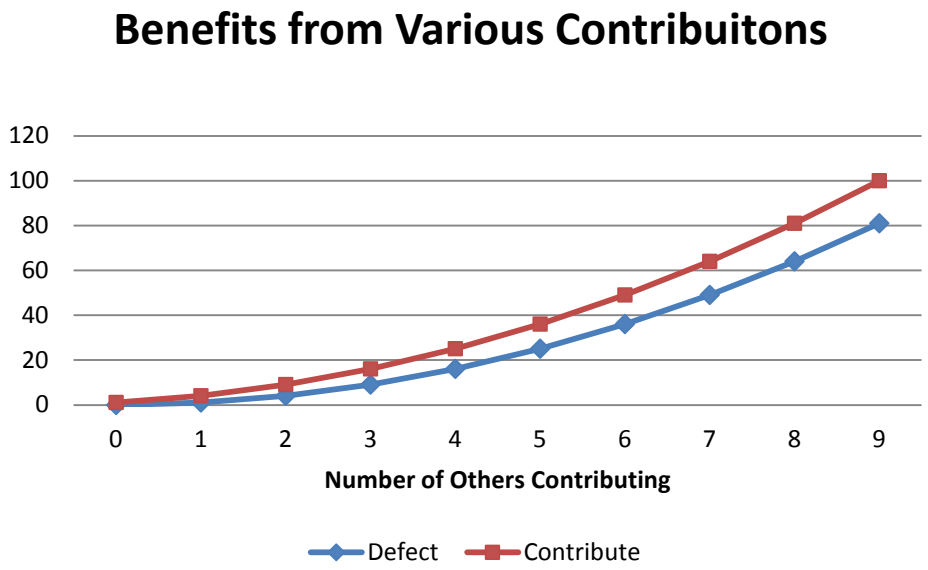
		A			B	
Number of others		**Benefits**			**Payoff**	
contributing		**Defect**	**Contribute**		**Defect**	**Contribute**
0		0^2	(0+1)^2		A-0	B-6
1		1^2	(1+1)^2		A-0	B-6
2		2^2	(2+1)^2		A-0	B-6
3		3^2	(3+1)^2		A-0	B-6
4		4^2	(4+1)^2		A-0	B-6
5		5^2	(5+1)^2		A-0	B-6
6		6^2	(6+1)^2		A-0	B-6
7		7^2	(7+1)^2		A-0	B-6
8		8^2	(8+1)^2		A-0	B-6
9		9^2	(9+1)^2		A-0	B-6

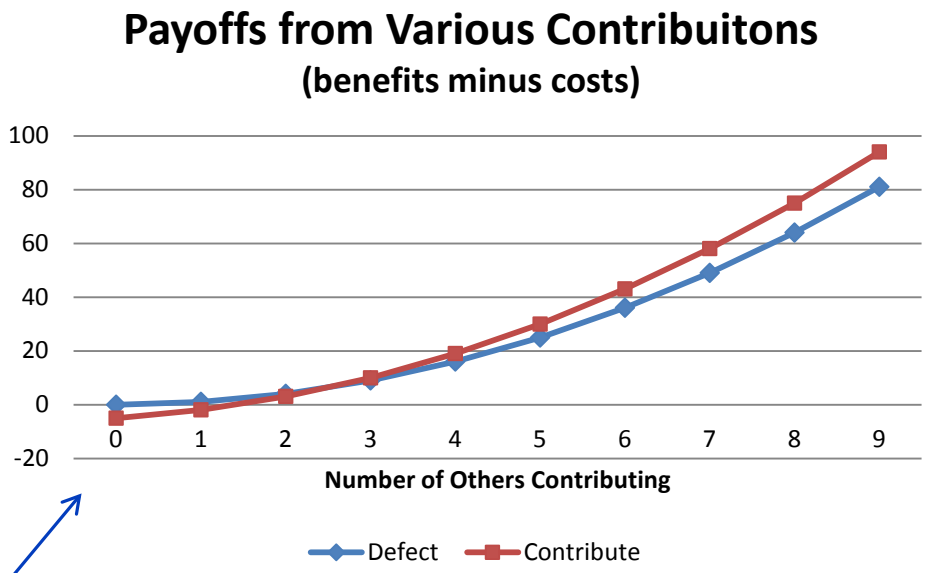
Assurance Game

- Drain a Meadow.

Number of others		Benefits			Payoff	
contributing		Defect	Contribute		Defect	Contribute
0		0.00	1.00		0.00	-5.00
1		1.00	4.00		1.00	-2.00
2		4.00	9.00		4.00	3.00
3		9.00	16.00		9.00	10.00
4		16.00	25.00		16.00	19.00
5		25.00	36.00		25.00	30.00
6		36.00	49.00		36.00	43.00
7		49.00	64.00		49.00	58.00
8		64.00	81.00		64.00	75.00
9		81.00	100.00		81.00	94.00

Assurance Game





Note: If 3 or more people contribute, it is better to contribute.
If less than 3 contribute, it is better to defect.

Assurance Game: normal form

Number of Others Contributing

Row		9	…	4	3	2	1	0
	contribute	10^2-6		5^2-6	4^2-6	3^2-6	2^2-6	1^2-6
	defect	9^2		4^2	3^2	2^2	1^2	0^2

Row		9	…	4	3	2	1	0
	contribute	94		19	10	3	-2	-5
	defect	81		16	9	4	1	0

Assurance Game: normal form

Number of Others Contributing

Row

	9	…	4	3	2	1	0
contribute	10^2-6		5^2-6	4^2-6	3^2-6	2^2-6	1^2-6
defect	9^2		4^2	3^2	2^2	1^2	0^2

Row

	9	…	4	3	2	1	0
contribute	(94)		(19)	(10)	3	-2	-5
defect	81		16	9	(4)	(1)	(0)

- Determine best responses for Row.
- If 2 or less people contribute, it is better to defect.
- If 3 or more people contribute, it is better to cooperate.
- Because everyone thinks this way, there are two N.E.

Chicken Game

- Ex: Defeating Somali Pirates. Benefits increase with each additional frigate, but three frigates fully do the job.

  - Benefits

    For 0 to 3 frigates, #frigates*2;

    For 4 or more frigates, 6+Ln(#frigates-1.28).

  - Costs

    1 for contributing and 0 for defecting.

  - Payoff

    benefits – costs.

Chicken Game

- Somali Pirates.

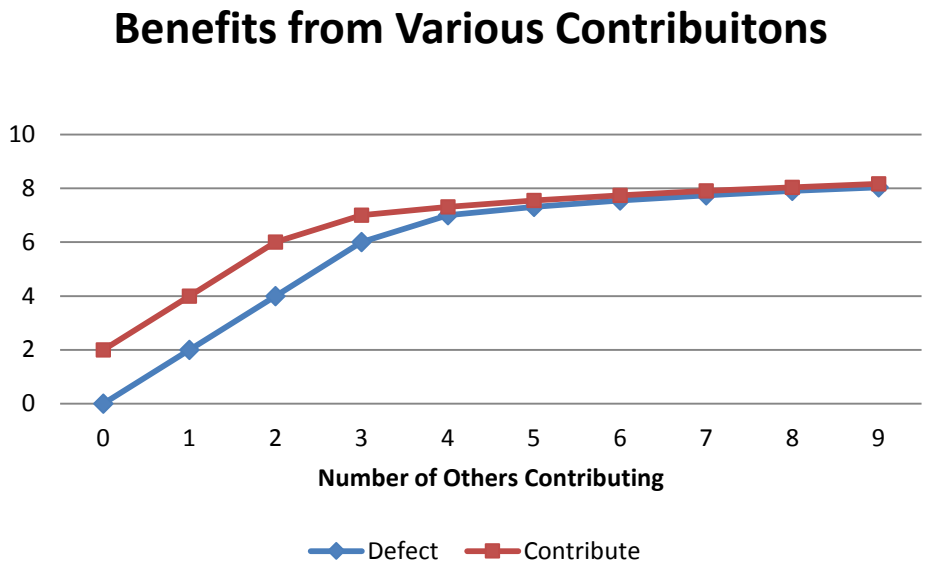
		A			B	
Number of others		**Benefits**			**Payoff**	
contributing		**Defect**	**Contribute**		**Defect**	**Contribute**
0		0*2	1*2		A-0	B-1
1		1*2	2*2		A-0	B-1
2		2*2	3*2		A-0	B-1
3		3*2	6+Ln(2.72)		A-0	B-1
4		6+Ln(2.72)	6+Ln(3.72)		A-0	B-1
5		6+Ln(3.72)	6+Ln(4.72)		A-0	B-1
6		6+Ln(4.72)	6+Ln(5.72)		A-0	B-1
7		6+Ln(5.72)	6+Ln(6.72)		A-0	B-1
8		6+Ln(6.72)	6+Ln(7.72)		A-0	B-1
9		6+Ln(7.72)	6+Ln(8.72)		A-0	B-1

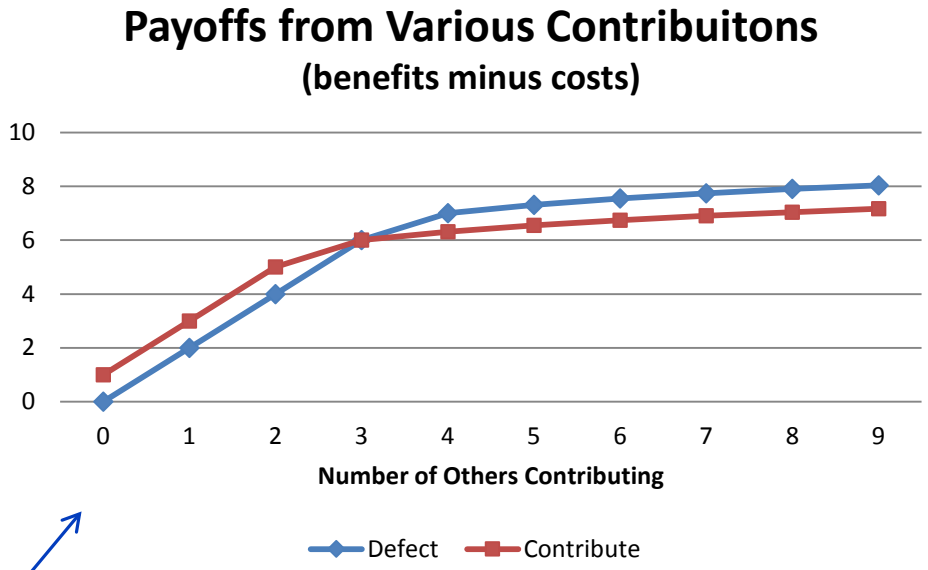
Chicken Game

- Somali Pirates.

Number of others		Benefits			Payoff	
contributing		Defect	Contribute		Defect	Contribute
0		0.00	2.00		0.00	1.00
1		2.00	4.00		2.00	3.00
2		4.00	6.00		4.00	5.00
3		6.00	7.00		6.00	6.00
4		7.00	7.31		7.00	6.31
5		7.31	7.55		7.31	6.55
6		7.55	7.74		7.55	6.74
7		7.74	7.91		7.74	6.91
8		7.91	8.04		7.91	7.04
9		8.04	8.17		8.04	7.17

Chicken Game





Note: If less the 3 people contribute, it is better to contribute.
If more than 3 contribute, it is better to defect.

Chicken Game: normal form

Number of Others Contributing

Row

	9	…	4	3	2	1	0
contribute	6*Ln(8.72)-1		6*Ln(3.72)-1	6*Ln(2.72)-1	(3*2)-1	(2*2)-1	(1*2)-1
defect	6*Ln(7.72)		6*Ln(2.72)	3*2	2*2	1*2	0*2

Row

	9	…	4	3	2	1	0
contribute	7.17		6.31	6	5	3	1
defect	8.04		7	6	4	2	0

Chicken Game: normal form

Number of Others Contributing

Row

	9	…	4	3	2	1	0
contribute	6*Ln(8.72)-1		6*Ln(3.72)-1	6*Ln(2.72)-1	(3*2)-1	(2*2)-1	(1*2)-1
defect	6*Ln(7.72)		6*Ln(2.72)	3*2	2*2	1*2	0*2

Row

	9	…	4	3	2	1	0
contribute	7.17		6.31	(6)	(5)	(3)	(1)
defect	(8.04)		(7)	(6)	4	2	0

- Determine best responses for Row.
- If less than 3 people contribute, it is better to cooperate.
- If more than 3 people contribute, it is better to defect.
- Exactly 3 frigates contributed is the N.E.

Summary

1. Prisoners' Dilemma
   - Equilibrium: no one contributes.
2. Assurance
   - Two equilibria: 1) no one contributes & 2) everyone contributes.
3. Chicken
   - Some contribute, some free-ride in equilibrium (intersection).