# THE MEDIAN VOTER THEOREM (ONE DIMENSION)

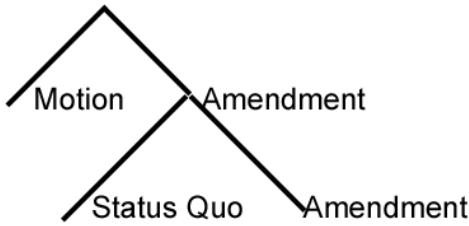
# Extra Credit

- Each of you will be assigned an "ideal point" between 1 and 10 inclusive (randomly drawn out of a hat).
- We will vote on alternatives between 1 and 10 inclusive using majority rule.
  - When we start, there will be no alternative on the floor (no one gets any points if there is no alternative on the floor when we adjourn).
  - Anyone can propose a number between 1 and 10. If someone seconds that proposal, we will vote on the proposal vs the number on the floor.
  - The alternative that receives a majority of votes becomes the number on the floor in the next round.
  - Voting concludes when someone motions adjournment, the motion is seconded, and a majority of players vote to adjourn. That number on the floor wins.
- Payoffs. You will receive $10 - (|x - i|)$ points on homework 3, where x is the winning number and i is your ideal point.
- In other words, the closer the final outcome is to your ideal point, the more points you receive.

Write your name on your number and give it to me.

# Single Dimensional Spatial Model

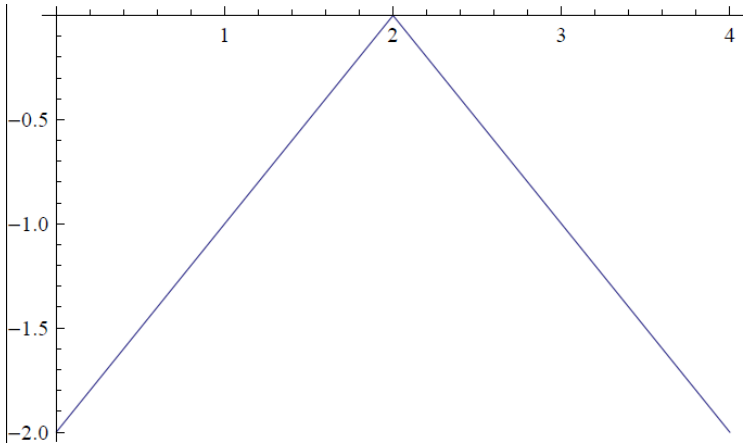
- Alternatives are the set of points on a line
  - Ideological spectrum
  - Spending on different programs
  - etc.
- Single-peaked preferences
  - Preferences are **satiable**
  - Each agent has an **ideal point** (most-preferred alternative)
- Symmetric preferences
  - Individuals prefer alternatives closer to their ideal point more that those farther away.
- Pairwise majority rule
  - Alternatives will be considered two at a time using majority rule.



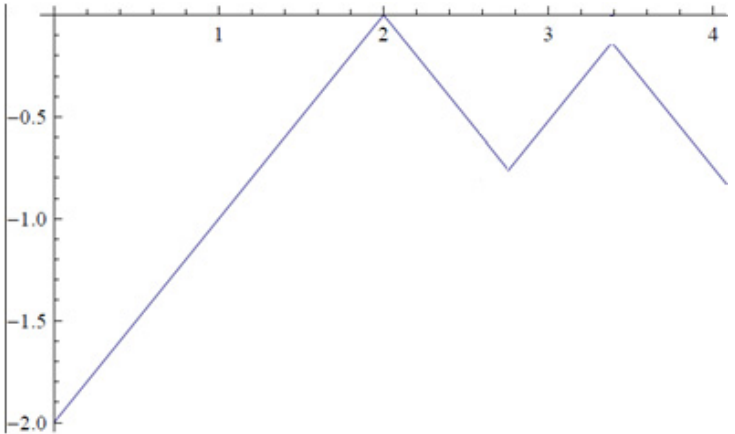
# Single Peaked Preferences

With single peaked preferences, utility is a decreasing function of the distance between the alternative and the ideal point.

- Single peaked preferences



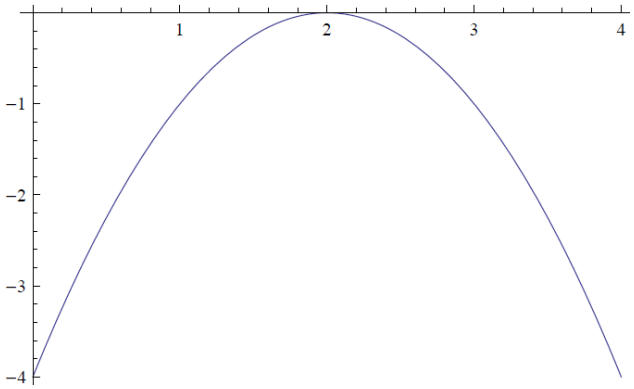

- Non-single peaked preferences



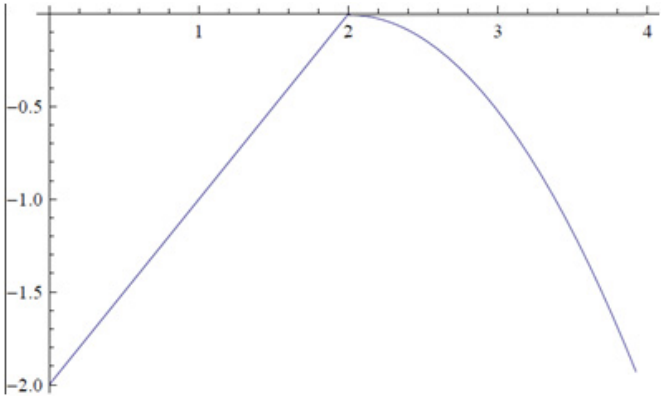
# Symmetric Preferences

With symmetric preferences, individuals prefer alternatives closer to their ideal point more than alternatives farther away.

- Symmetric preferences



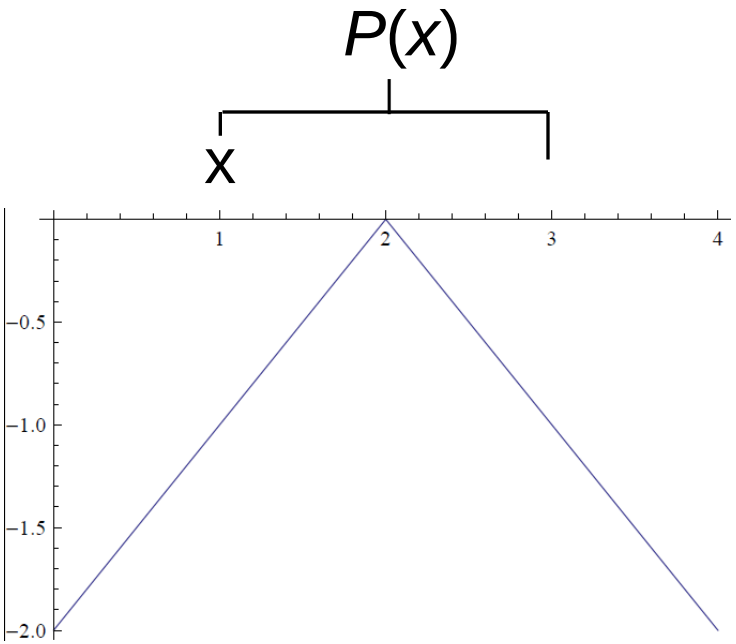

- Asymmetric preferences



# Preferred-to set

The **preferred-to set** of x is the set of points y such that yPx

$$P(x) = \{y \in X \mid U(y) > U(x)\}$$



# Example

Suppose policy is one-dimensional and that a legislator has single-peaked and symmetric preferences with an ideal point at 4. If the status quo policy is located at 7, what is the set of policies that the legislator prefers to the status quo?

# Equilibrium Concept

- **Core**: Alternative x is an element of the core of an *f*-voting rule game if there does not exist another alternative y that *f* individuals prefer to x.
  - Ex: x is an element of the majority rule core if there does not exist another alternative y that a majority of individuals prefer to x.
  - The core is an equilibrium concept for spatial voting games.

# Median Voter Theorem

Given:

1) n > 2 (and n is odd),
2) pairwise majority rule voting,
3) alternatives are on a single dimension,
4) preferences are single peaked.

**MVT**. The core is the median voter's ideal point under pairwise majority rule (i.e. the median voter's position is in equilibrium).

Remarks

- Also works for n even, you just have to understand what we mean by "median."
- Median voter = individual such that half of the other ideal points are opposite sides of the median's ideal point.
- Called (Duncan) Black's Median Voter Theorem.

# Proof of The Median Voter Theorem (*n* odd)

Notation

$t_m$ = median's ideal point

q = the status quo.

L = (n-1)/2 number of ideal points to the left of $t_m$

R = (n-1)/2 number of ideal points to the right of $t_m$

Assume $q = t_m$. First show that q is in the core.

Consider an arbitrary x such that $x < t_m$. Note that $R \cup \{t_m\}$ individuals prefer q to x; thus, a majority do not not prefer x to q.

Consider an arbitrary y such that $y > t_m$. Note that $L \cup \{t_m\}$ individuals prefer q to x; thus, a majority cannot not prefer y to q.

The proof follows by showing that any $z \neq t_m$ is *not* in the core, which follows because q will attain either $R \cup \{t_m\}$ votes or $L \cup \{t_m\}$ votes, and defeat z.

# Proof of The Median Voter Theorem (*n* even)

Prove the MVT for n even.

- Order the voters ideal points from smallest to largest and note that the median pivots are in position $M_1 = n/2$ and $M_2 = (n+2)/2$. The total number of voters to the right (larger) than $M_2$ are $n - (n+2)/2 = (n - n/2) - 1 = n/2 - 1$. This means that there is less than a majority of the members to the right of $M_2$ (larger than $M_2$). Hence, any alternative to the right of $M_2$ (larger) cannot receive majority of votes in favor of it.
- Similar reasoning shows that there is not a majority of individuals to the left (smaller) than $M_1$.
- Hence, for n even, the core is $[M_1, M_2]$.
  - EX: on board.

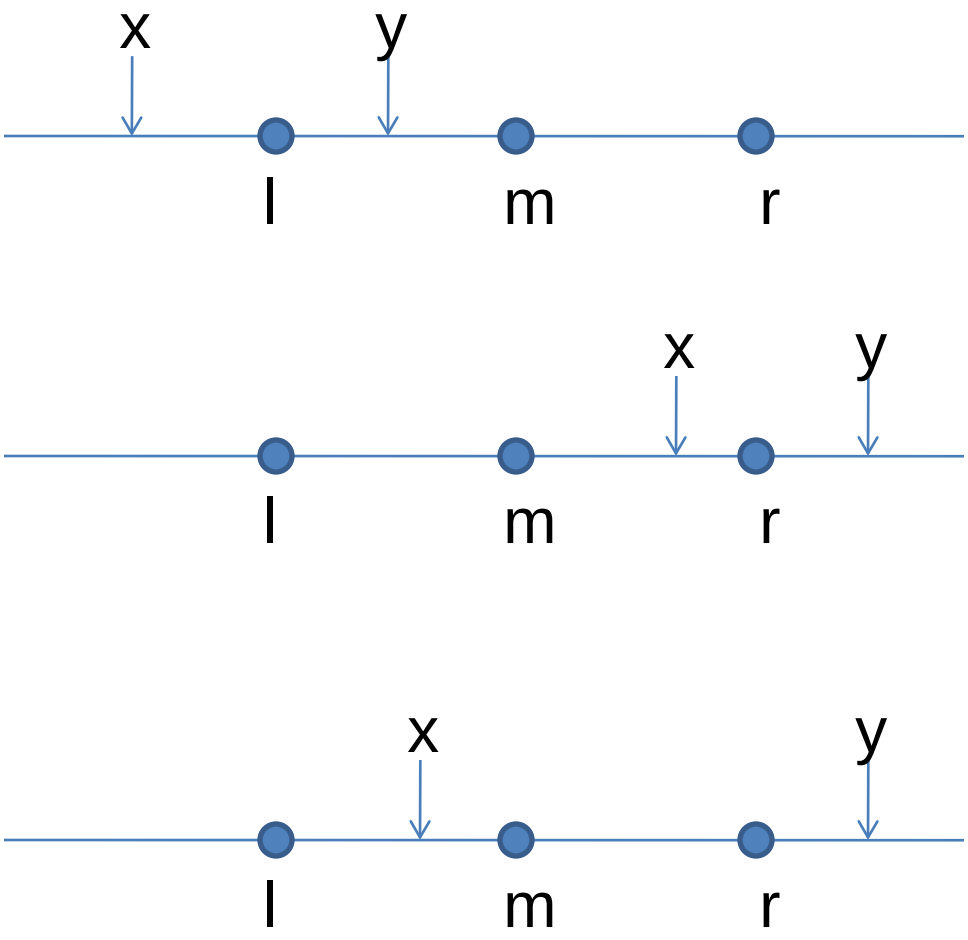
# Corollary of the MVT

Given:

1) n > 2 (and n is odd),
2) Pairwise majority rule voting,
3) alternatives are on a single dimension,
4) preferences are single peaked,
5) and preference are symmetric.

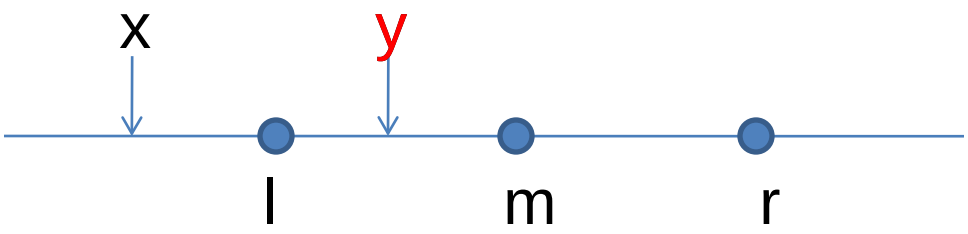
    1. Corollary to the MVT: alternatives closer to the median voter will defeat alternatives farther from the median voter under pairwise majority rule.

# Sketch of proof of the Corollary




Note: the alternative closer to the median gets the median's vote and half the voters to one side. That's why the closer alternative always wins.

# Because of the corollary, alternatives will be drawn toward the median.



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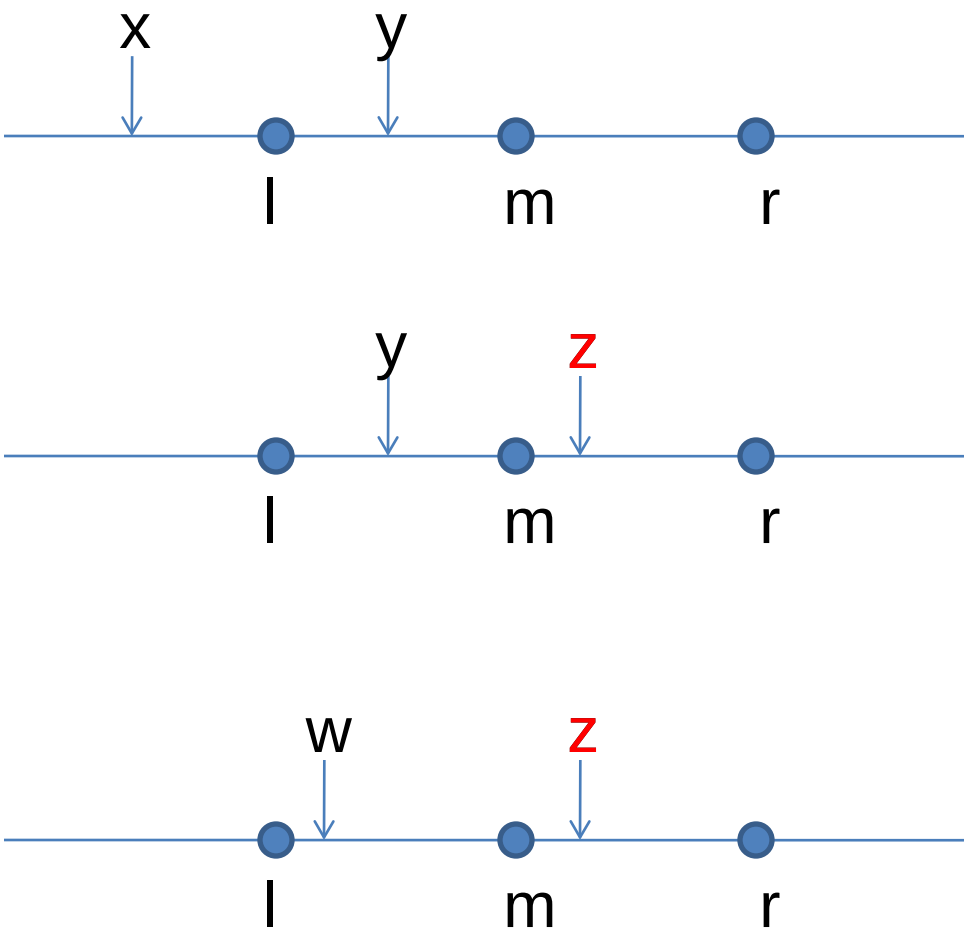


# Other Results

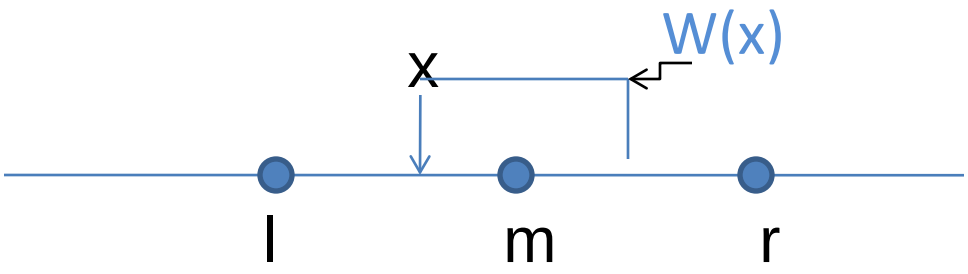
Given:

1) n > 2 (and n is odd),
2) pairwise majority rule voting,
3) alternatives are on a single dimension,
4) preferences are single peaked,
5) and preference are symmetric.

2. The median voter's ideal point is the core (i.e. in equilibrium) – as before.
3. The social preference ordering is the same as the median voter's preference ordering.
4. Social preferences created by majority rule are transitive.

# Win sets

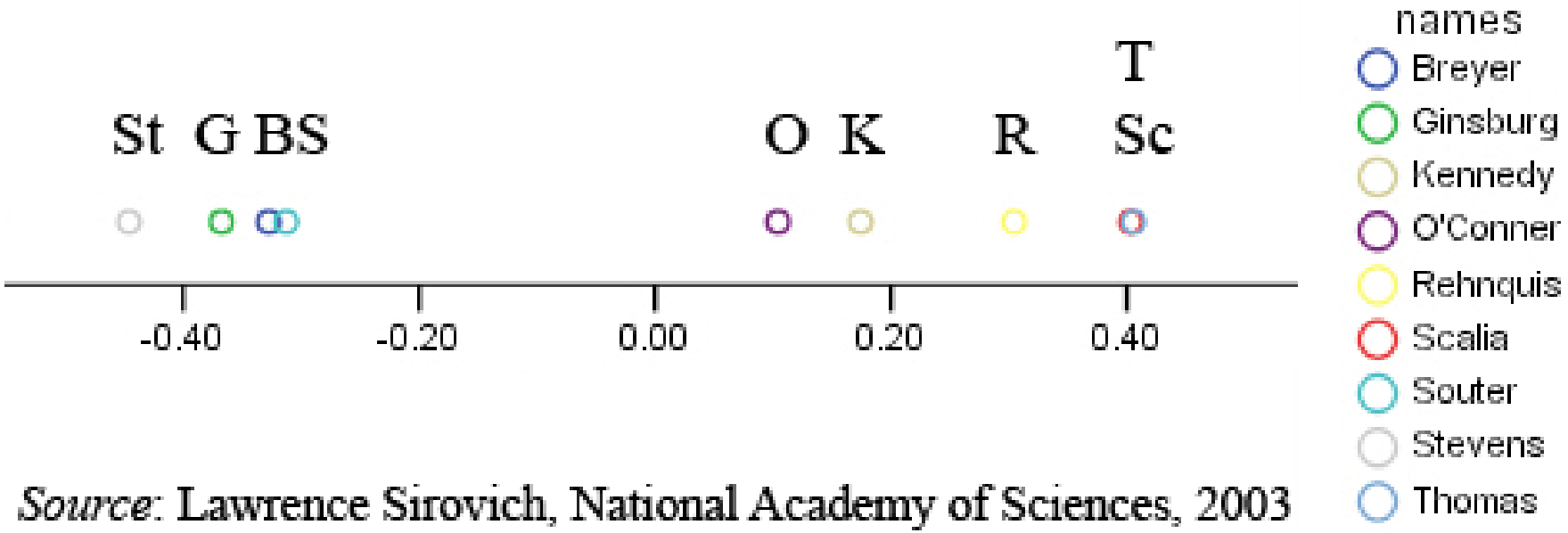
Let the majority rule "win set" of x be the set of alternatives that a majority prefers to x: W(x) = {y | yPx}




The win set is the set of alternatives that can pass – which helps us study status quos that are not in equilibrium.

# Application: Supreme Court

- US Supreme Court (1995-2002)




*Source*: Lawrence Sirovich, National Academy of Sciences, 2003

  - What happened when we replaced Rehnquist with Roberts?
  - What happened when we replaced O'Conner with Alito?
  - What happened when we replaced Souter with Sotomayor?
  - What happened when we replaced Stevens with Kagan?

# Application: Legislative chambers and committees

- Assume an odd number of members on the floor and in committee.
- Preferences are single-peaked and symmetric.
- The legislature and committee each use simple majority rule.

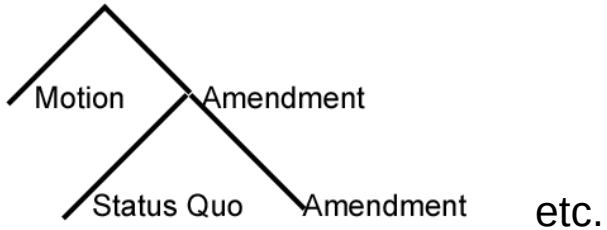
Implication

- Median voter results imply the analysis can be simplified by considering only the median of the legislature and the median of the committee.

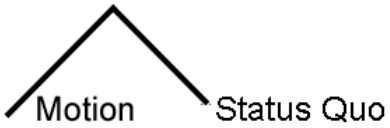
# Application: Legislative chambers and committees

## The effect of open vs. closed rules in Congress

- Open: allows any amendment germane to a bill to be proposed on the floor.




b. Closed Rule: prohibits amendments on the floor.

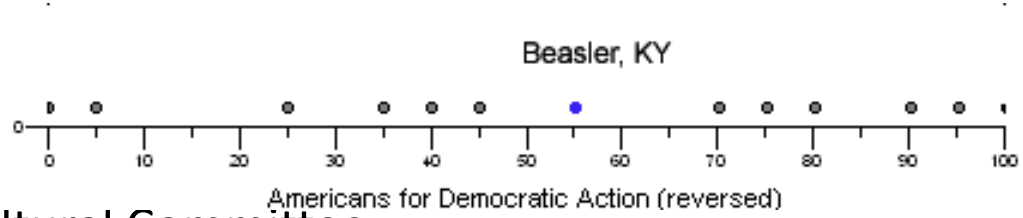


**Membership on House Agricultural Subcommittees Dealing with Tobacco**

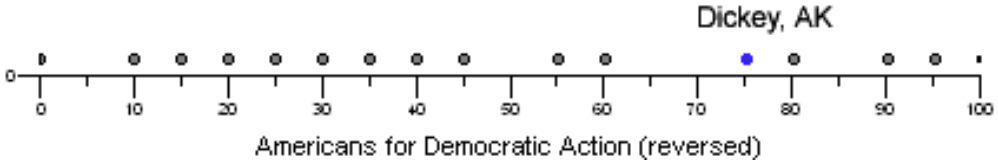
| Subcommittee on Tobacco, 1973 | Subcommittee on Tobacco and Peanuts, 1983 | Subcommittee on Specialty Crops & Natural Resources, 1993 |
|---|---|---|
| **Stubblefield, D-KY** | **Rose, D-NC** | **Rose, D-NC** |
| Jones, D-NC | Jones, D-NC | Baesler, D-KY |
| Mathis, D-GA | Hatcher, D-GA | Bishop, D-GA |
| Rose, D-NC | Thomas, D-GA | Brown, D-CA |
| Litton, D-MO | Whitley, D-NC | Condit, D-CA |
| Mizell, R-NC | Tallon, D-SC | Clayton, D-NC |
| Wampler, R-VA | English, D-OK | Thurman, D-FL |
| Madigan, R-IL | Stenholm, D-TX | Minge D-MN |
| Young, R-SC | Hopkins, R-KY | Inslee, D-WA |
| | Roberts, R-KS | Pomeroy, D-ND |
| | Skeen, R-NM | English, D-OK |
| | Franklin, R-MS | Senholm, D-TX |
| | | Peterson, D-MN |
| | | Lewis, R-FL |
| | | Emerson, R-MO |
| | | Doolittle, R-CA |
| | | Kingston, R-GA |
| | | Goodlatte, R-VA |
| | | Dickey, R-AK |
| | | Pombo, R-CA |

Source: Loomis, *The Contemporary Congress*, Table 5-2.

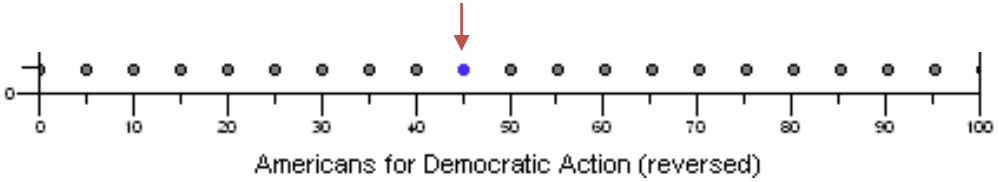
– Subcommittee on Specialty Crops




– Agricultural Committee



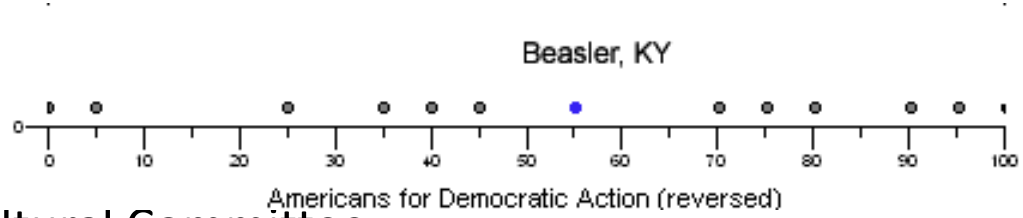

– House Floor




If an open rule was allowed on the floor, what do you predict would be the outcome of a bill coming out of this committee after it was amended by the floor?

a. The floor median.

– Subcommittee on Specialty Crops

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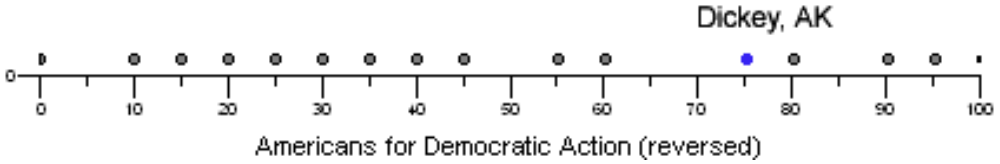


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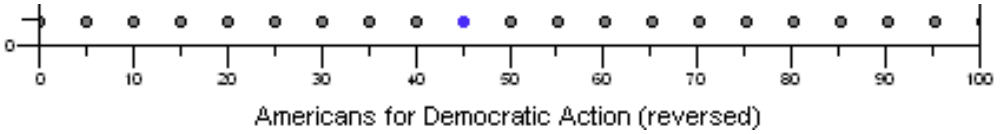
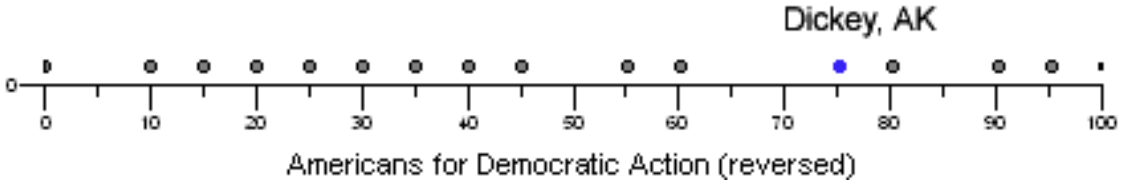
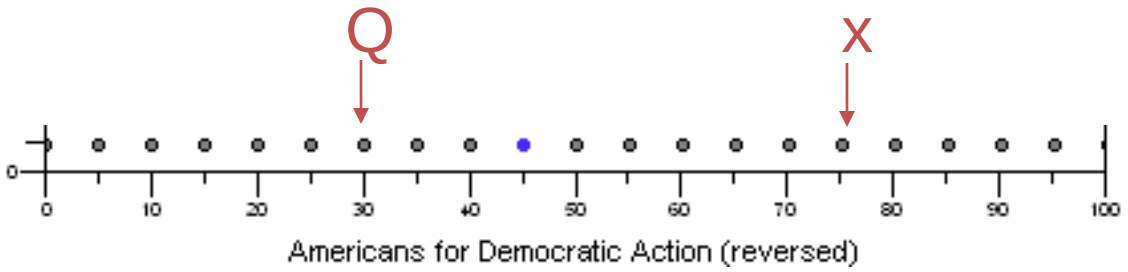



What if the bill was voted on using a closed rule?

– Agricultural Committee




2. House Floor

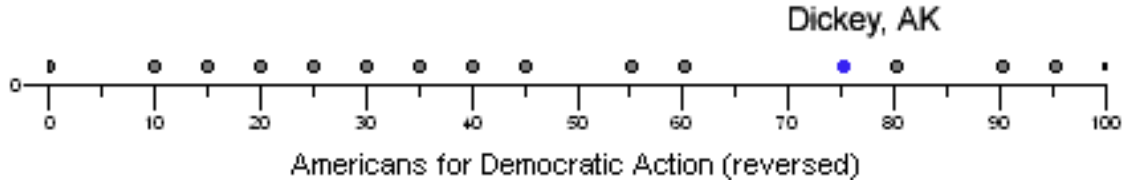



To make this simple, I'm going to get rid of the sub-committee for the moment.

Now suppose the status quo, Q = 30, the committee proposes x =75, and the floor votes under a closed rule. What should be the outcome?

a. Q wins. Under a closed rule, the choice is between x and Q. Since Q is closer to the floor median, Q wins.

– Agricultural Committee

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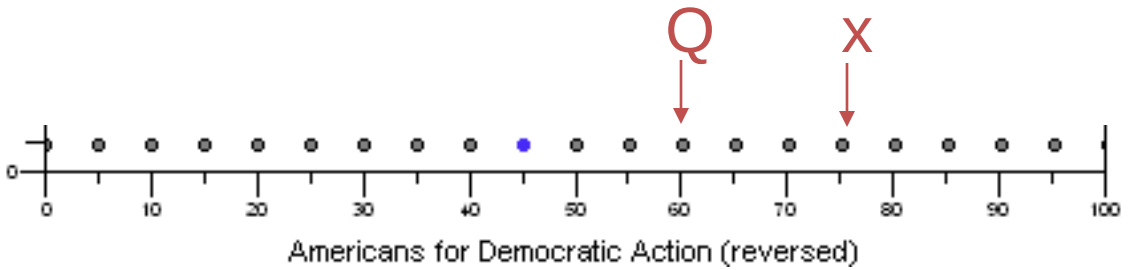
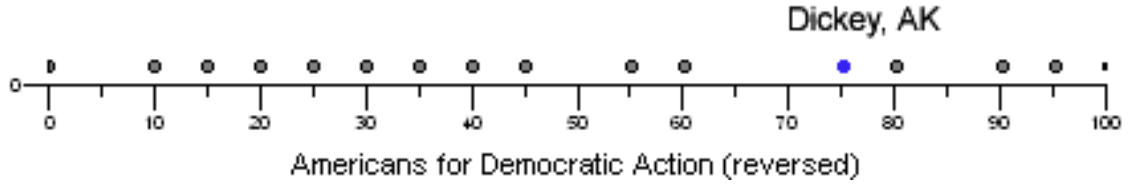



Now suppose the status quo, Q = 60, the committee proposes x =75, and the floor votes under a closed rule. What should be the outcome?

a. Q wins. Under a closed rule, the choice is between x and Q. Since Q is closer to the floor median, Q wins.

– Agricultural Committee

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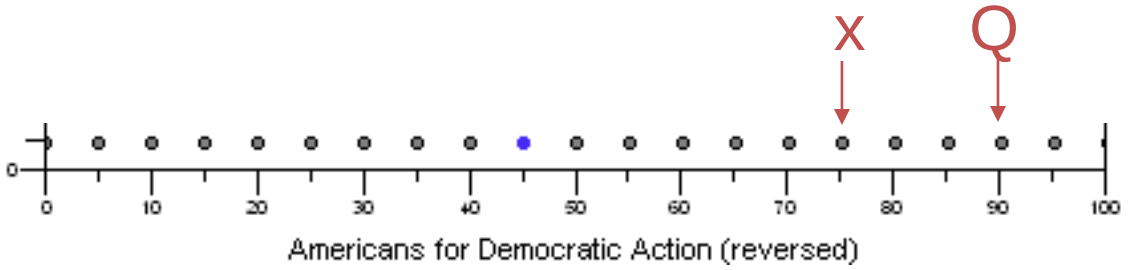


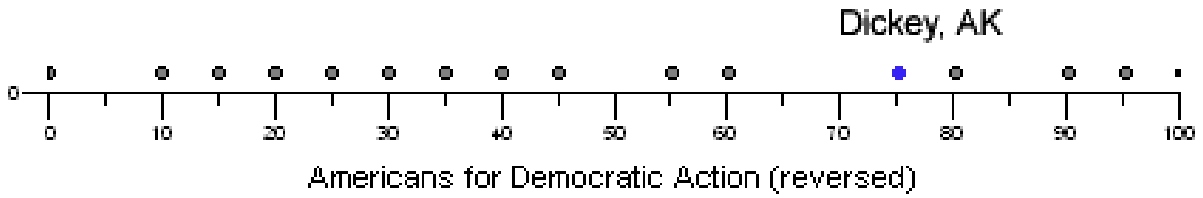

Now suppose the status quo, Q = 90, the committee proposes x =75, and the floor votes under a closed rule. What should be the outcome?

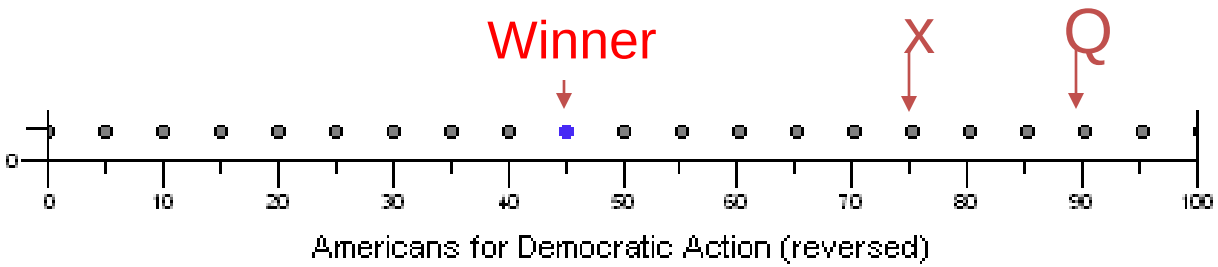
a. x=75 wins. Under a closed rule, the choice is between x and Q. Since x is closer to the floor median, x wins.

What if the vote was under an open rule?

– Agricultural Committee

2. House

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a. x=75 wins. Under a closed rule, the choice is between x and Q. Since x is closer to the floor median, x wins.

What if the vote was under an open rule?

# Summary

- Under an open rule, the floor median might be the expected outcome.
  - This makes the floor powerful, the committee not.
- Under a closed rule, either the status quo or the committee proposal should win (depending upon which alternative is closer to the floor median).
  - This makes the committee relatively powerful.

The House is often using closed rules. What does this say about where the rules committee would like the power to reside: committee or floor?