POLS 8000
Introduction to Rational Choice
Dougherty

Homework 3:
Incomplete Information Games & N-Player Games
(due: in class, March 26)

Directions: The following questions should help you understand the material in the first part of the course. Please write your answers *neatly* on a separate sheet of paper. You will probably have to re-write your answers before you turn them in. Furthermore:

1) You are welcome to ask background and clarification questions from other students in the class, but you must attempt to work through as much of the homework as possible on your own, and then work with other students only when you are stuck or want to check your answers.
2) You need to write up your own answers, using your own words and explanations. If you turn in the exact same answer as another student, I will consider it academic dishonesty.
3) Every answer needs an explanation which includes how you derived the solution. Please justify your response, include requisite math, and **SHOW YOUR WORK**.
4) Late homeworks will be reduced one letter grade for every working day they are late and will not be accepted after the next class starts (folks want to see the answers).
5) Each sub-question is worth 10 pts.

1. A husband (player 1) and a wife (player 2) face a modified version of the Battle of the Sexes game, which includes asymmetric information and sequential play. In this game, the wife knows the husband has forgotten whether she prefers going to the movies or going to the ballet. Both know the husband prefers the ballet. Rather than moving simultaneously, the husband will chose first in this version of the game. If the wife is type "Ballet," they will receive (3, 3) from both choosing the ballet, (1, 1) from both choosing the movies, and (0, 0) from one of them going to the ballet and the other going to the movies – where (x, y) are the payoffs to the husband and wife respectively. If the wife is type "Movies," they will receive (1, 1) from both choosing the ballet, (1, 3) from choosing the movies, and (0, 0) from one of them going to the ballet and the other going to the movies – where again (x, y) are the payoffs to the husband and wife respectively.

a. Using the information stated above, create a sequential, asymmetric information game of the couple's decision to go to the ballet or the movies. For this game make *p* the probability that the wife prefers ballet [hint: this is similar to the game for the Cuban Missel Crisis].

b. Solve the game using a combination of backward induction and expected value. What are the sequential equilibria of this game? [hint: first determine the values of *p* for which the husband should go to the ballet and the values of *p* for which he should go to the movies].

2. There are 100 *rational* individuals in a small town trying to decide whether to vote in the next referendum about whether they should fire the sheriff. Voting yea on the referendum is a vote to fire the sheriff. Voting nay is a vote to keep the sheriff. *The sheriff is fired if and only if more individuals vote in favor of removing the sheriff than individuals who vote to remove the sheriff (i.e. yeas exceed nays, with ties going to the status quo of keeping the sheriff)*. Each individual sees the decision to vote as a step-good game as follows:

	Number of Others Voting for i's Favorite Candidate		
	more than half (p_1)	exactly half (p_2)	less than half (p_3)
Vote	b − c + $	b − c + $	−c + $
Don't Vote	b	0	0

where: b = the benefit individual i receives from electing their favorite candidate (the candidate differential).

c = the cost of voting to individual i.

$ = the bribes received from individual i for the act of voting.

p_1 = the probability that the vote will be decided in favor of i's favorite outcome (with or without i's support).

p_2 = the probability that i's vote will be pivotal in the election.

p_3 = the probability that the vote will be decided against i's favorite outcome (with or without i's support).

Note: the exact value of each variable may differ by individual (partly because the relevant values are those that each individual perceives). There are no other costs or benefits.

a. For Alma (one of the individuals in the town), b = 150, c = 30, and p_2 = .02. At what values of $ is it rational for Alma to vote? Why?

b. Suppose that 100 people in the town can be classified into three groups based on their *perceived* benefits, costs and probabilities of being pivotal, as follows:
Group A (40 people): b = 200, c = 25, and p_2 = .01.
Group B (30 people): b = 100, c = 20, and p_2 = .01.
Group C (30 people): b = 150, c = 30, and p_2 = .02.
These numbers are fixed. If there are no bribes from the act of voting (i.e. $ = 0), how many people will vote? Why?

c. Assume the values of b, c, and p_2 are the same as stated in "question b." Now suppose that *everyone* in the town prefers to fire the sheriff rather than keep him. Since everyone wants to fire the sheriff should we expect the referendum to pass using simple majority rule?

d. Assume the values of b, c, and p_2 are the same as stated in "question b" and that everyone in the town prefers to fire the sheriff rather than keep him. Now Al Swearengen tries to bribe enough individuals to get the people to vote to fire the sheriff. If he wants to do this with the least amount of cost, whom should he bribe? Need he bribe anyone at all? Why? [note: assume Al is one of the many individuals in group A, B, or C. Don't treat him differently. Also assume complete information (for this question and those forward). In other words, he knows everyone's preferences, benefits, costs, and p_2].

e. Assume the values of b, c, and p_2 are the same as stated in "question b" and that people prefer to fire or keep the sheriff as follows: 50% of group A prefer firing the sheriff, 50% of group B prefer firing the sheriff, and 60% of group C prefer firing the sheriff. The other individuals prefer keeping the sheriff. Furthermore, Group A receives civic duty (D) benefits = 26, group B receives civic duty (D) benefits = 25, and group C receives civic duty (D) benefits = 25. Al realizes that he has only $100 available for bribes. If he wants to do this with the least amount of cost, whom should he bribe? Can he afford it? Why?

3. Ten hunters have to decide whether to spend the day hunting a badger or hunting a wildebeest. Successfully capturing a wildebeest requires at least six hunters to pursue the wildebeest together. Each hunter can capture a badger by themselves. More specifically, there are:

- 10 hunters, each of which can choose to hunt wildebeest (*W*) or hunt badger (*B*);
- capturing the wildebeest requires 6 or more hunters to choose *W*;
- If $k \geq 6$ hunters choose *W*, they divide the wildebeest into equal shares, such that everyone who hunted wildebeest gets $2+ 1/k$ worth of the meat (i.e., benefits equal $2+1/k$);
- An individual who hunts wildebeest incurs a cost of c;
- If an individual hunts a badger, he catches it and receives 2 units of benefit at a cost of 1 unit of effort;
- Any share of the wildebeest is more valuable than a full share of the badger, which explains why hunters get a payoff of $2+ 1/k$ if at least five of them choose *W*.

a. use the information above to create an *n*-player normal form game (hint: label the columns "*k* hunting wildebeest (10-*k* hunting badger)").

b. What is / are the Nash equilibria in this game if c = 1? What is / are the Nash equilibria in this game if c = 1.125? Show your work on a separate matrices and explain.

4. In the Cuban Missel Crisis game presented in class, and in your book, the United States received a payoff of -10 if the U.S. threatens and the Soviets defied (regardless of player type). Find the sequential equilibrium of the same game if we replace -10 with -12. Show all of your work [*hint*: lecture notes should help].