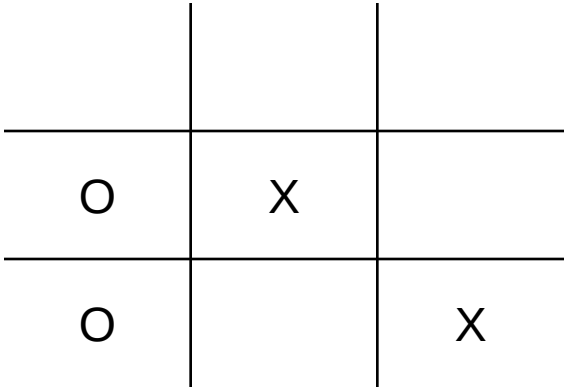
INTRODUCTION TO GAME THEORY

Game Theory

A. Tic-Tac-Toe (loser pays winner $5).

1. Are there good and bad moves in tic-tac-toe?



a. yes, at least some times.

b. def: action – something a player can choose at a particular point in the game.

Game Theory

2. Is there an optimal strategy to play?
   a. Def: strategy – a complete set of actions for a player in a game.
   b. Yes, bad moves (or actions) imply bad strategies.
3. If two people play optimal strategies, what's the outcome?
   a. A tie. …optimal does not mean win.
4. Should we expect two people to play to a tie?
   a. _?_.
   b. Def: preference – an individual's liking of one outcome compared to another (usually expressed in terms of payoffs).
   c. Def: rationality – an individual is rational if they chose the strategy that gets them their most preferred outcome (or more preferred outcome, if "most" preferred is not attainable).
      a. Rationality only means something with respect to an individual's pre-specified valuation of the outcomes.

Game Theory

Aside:

The probability of an event *A*, expressed as *P*(*A*), has the following properties:

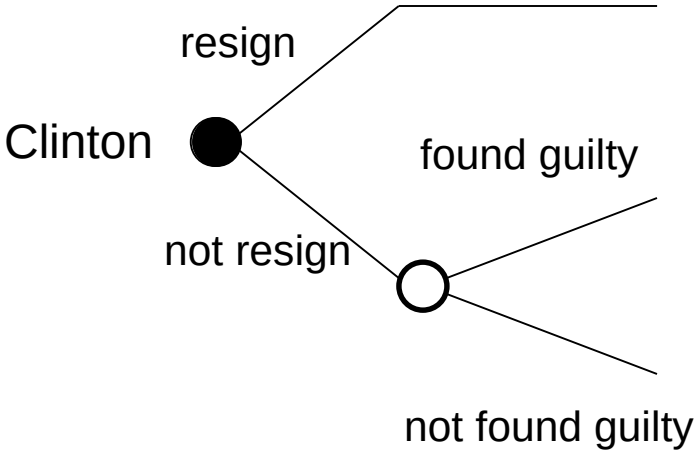
1. $0 \leq P(A) \leq 1$.
2. $P(A) = 1 - P(\sim A)$.
3. $P(\varnothing) = 0$.
4. For mutual exclusive and exhaustive events $A_1, A_2, \ldots A_n$:

$$\sum_i^n P(A_i) = 1$$

Game Theory

B. One Player Games (games against nature)

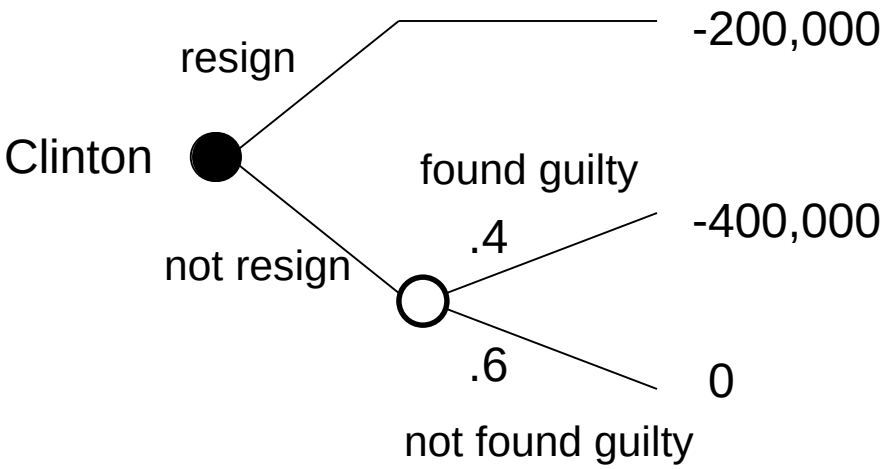
1. Bill Clinton Decides to Resign over Monica Lewinski.



Game Theory

B. One Player Games (games against nature)

1. Bill Clinton Decides to Resign over Monica Lewinski.



Therefore:
don't resign

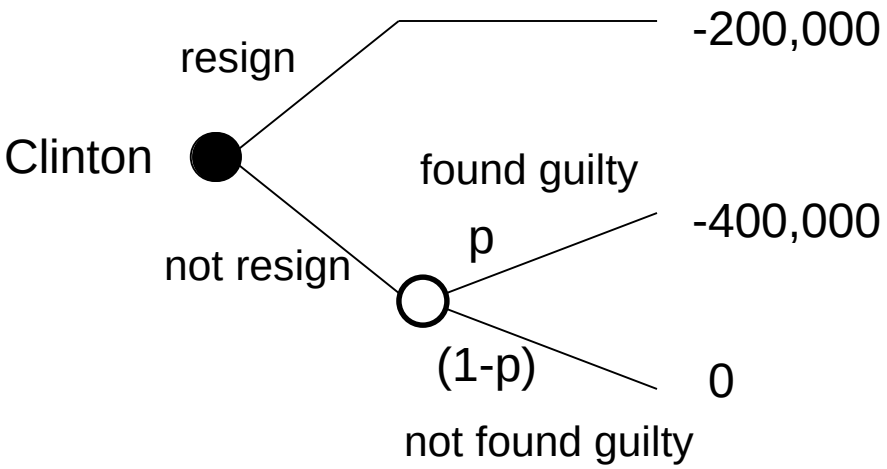
Calculate Expected Value:

Resign	-200,000 =	-200,000
Not Resign	.4(-400,000) + .6(0) =	-160,000

Game Theory

B. One Player Games (games against nature)

2. What if we didn't know the probabilities?



Sensitivty Analysis: Calculate Expected Value

Resign	-200,000
Not Resign	p(-400,000) + (1-p)0

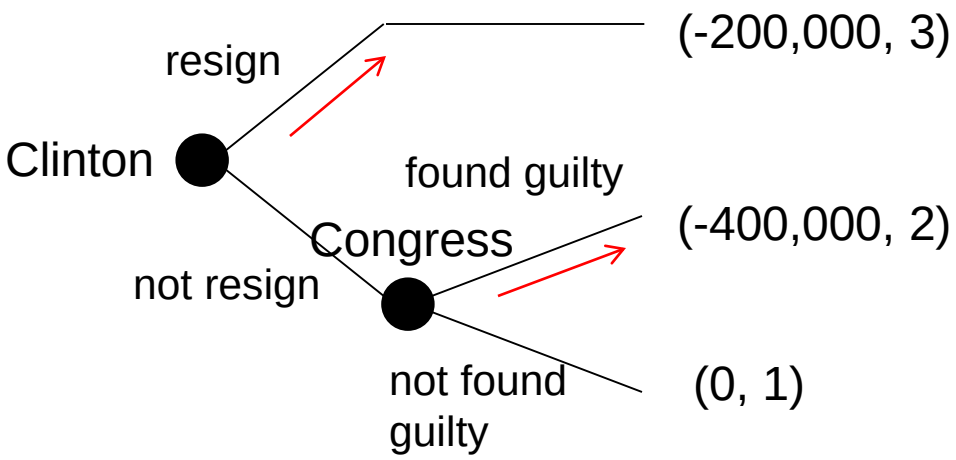
Resign if and only if -200,000 > p(-400,000)

½ < p

Game Theory

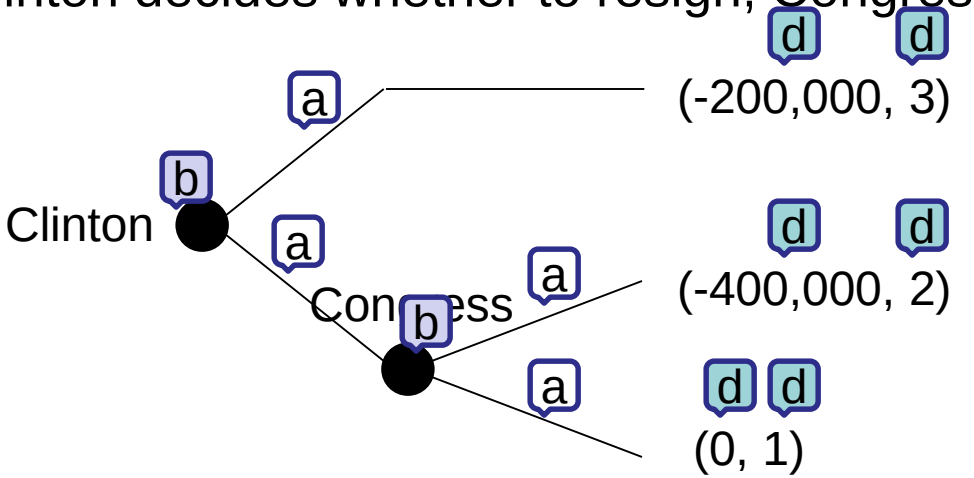
C. Multiple Player Games

1. Three types
    1. Zero sum games – your gains are my losses.
    2. Cooperative games – games where agreements are binding (skip).
    3. Non-cooperative games – games where coordination of strategy must be done through play alone.
        a) Zero sum games are a special type of non-cooperative game.
2. Ex: Clinton decides whether to resign, Congress responds.



Game Theory

2. Ex: Clinton decides whether to resign, Congress responds.



More definitions

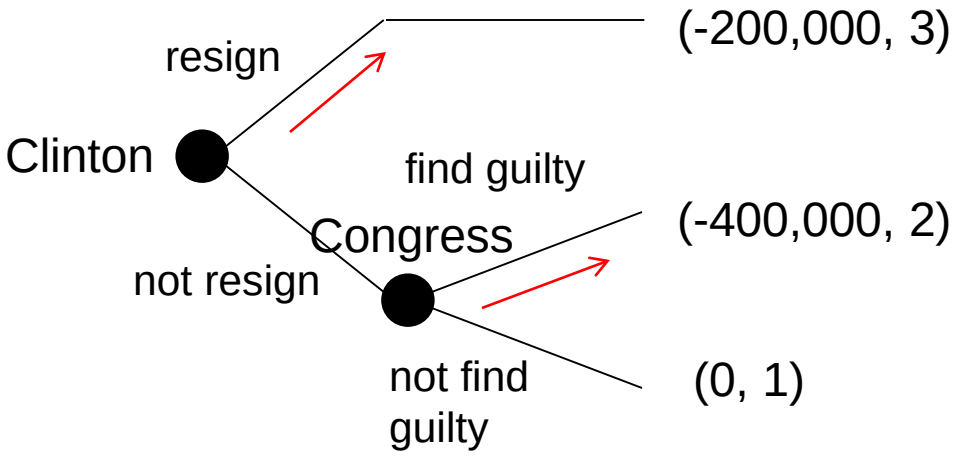
a. Branches – the actions that can be chosen at each decision node (i.e. the lines).
b. Decision node – a point of decision for an actor.
c. Chance node – a point where mother nature moves (i.e. a probabilistic event occurs).
d. Payoff – a player's valuation for an outcome (also known as utility).

EXTENSIVE FORM (SEQUENTIAL MOVE) GAMES

Extensive Form Games

A. Backward Induction

1. Start at the back of the game, determine what is rational at each node, then work forward.
2. subgame perfect equilibrium (SPE) – the expected outcome of the game, determined by backward induction.

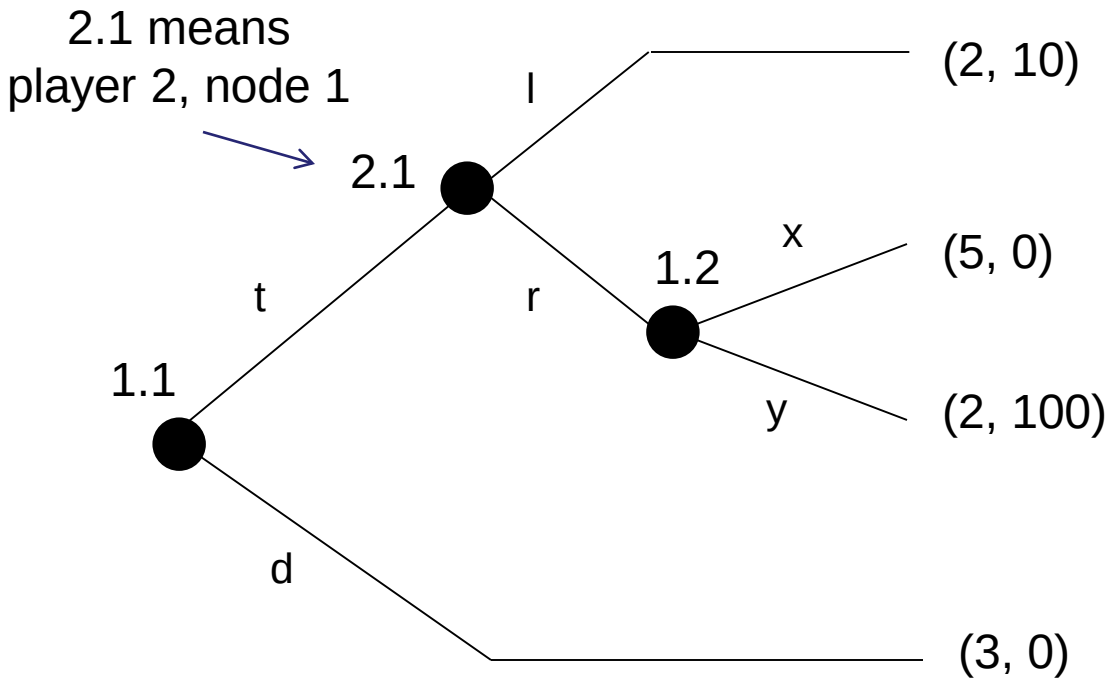


SPE = {resign; find guilty}

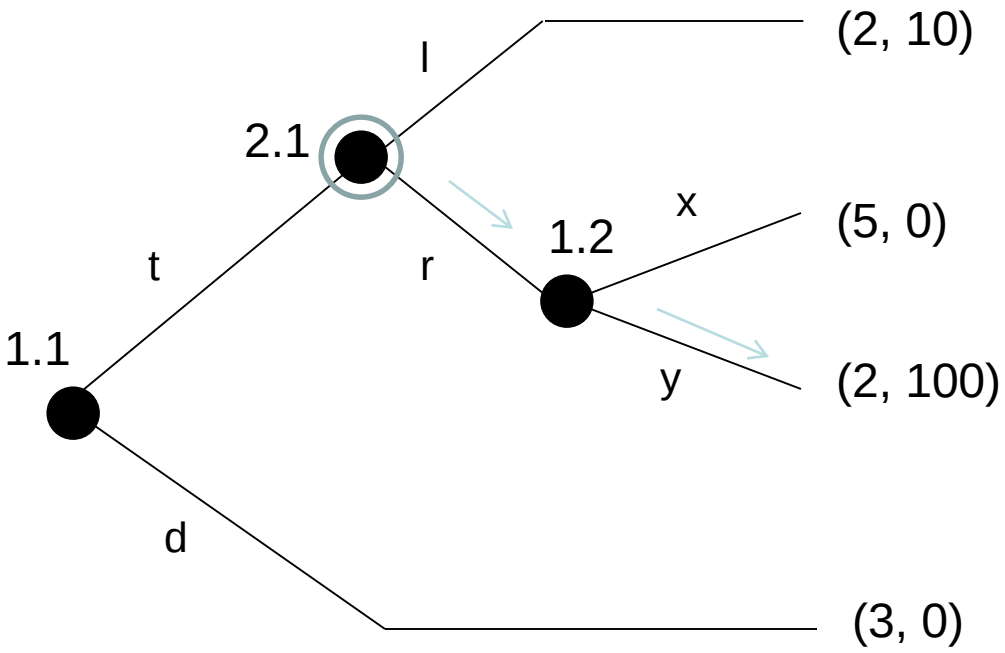
… always written first player's strategy; second player's strategy; etc.

Extensive Form Games

3. Another Example



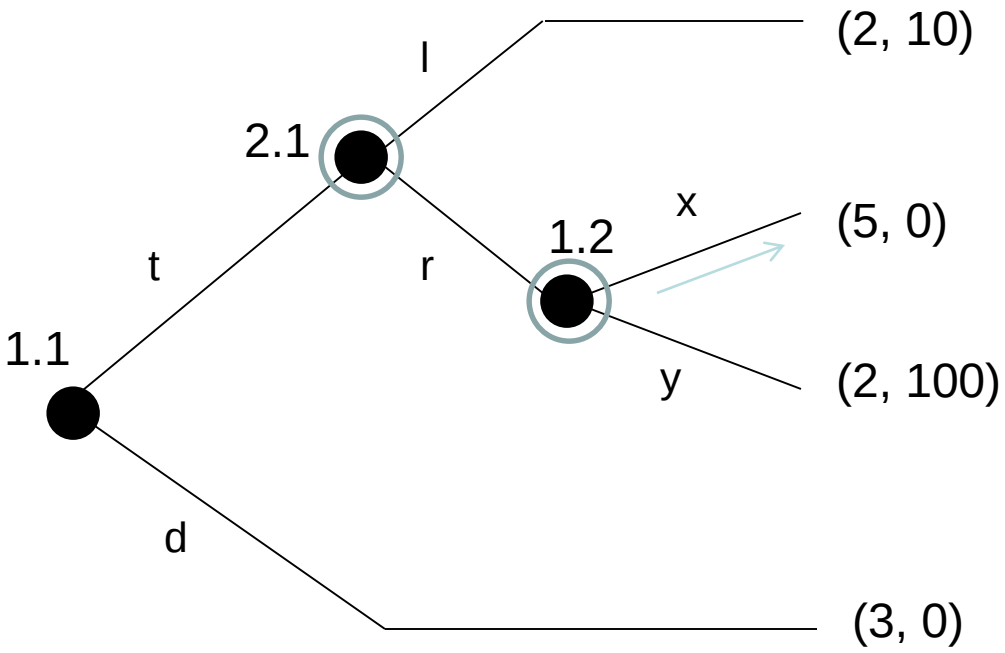
Extensive Form Games



What happens if we attempt forward induction?

Player 2 might play r at 2.1 because he thinks he could get 100.

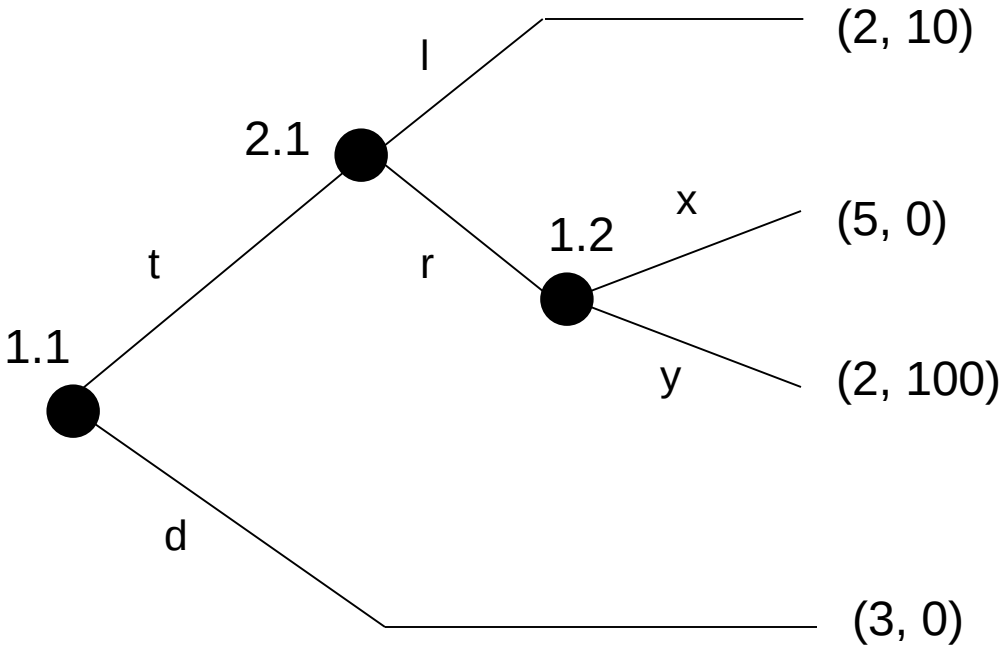
Extensive Form Games



What happens if we attempt forward induction?

Player 2 might play r at 2.1 because he thinks he could get 100.
But if he anticipates 1's move, he will know that 1 will never play y at 1.2.

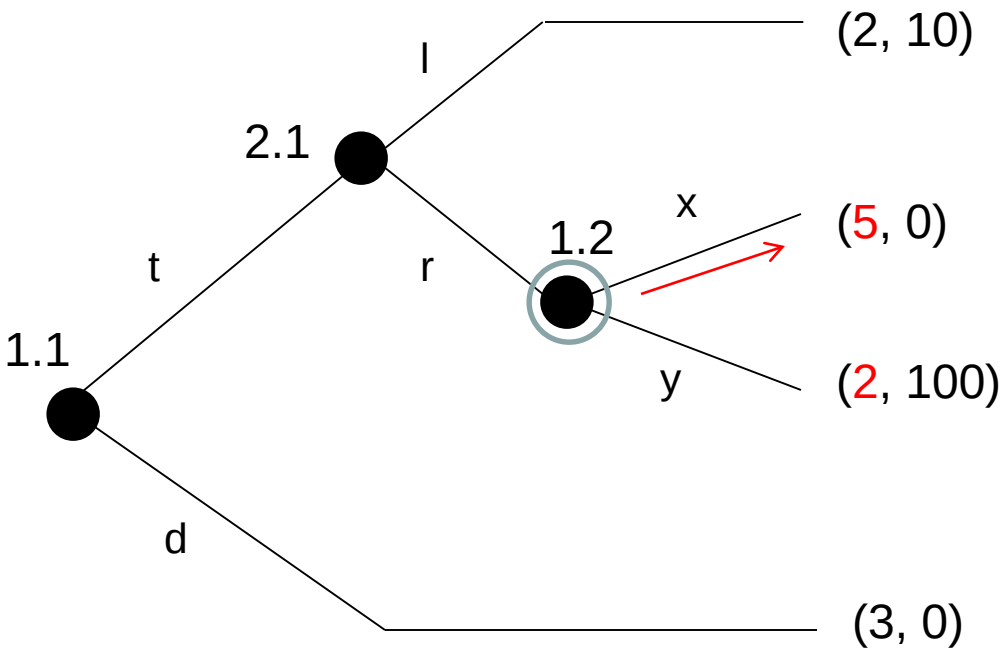
Extensive Form Games



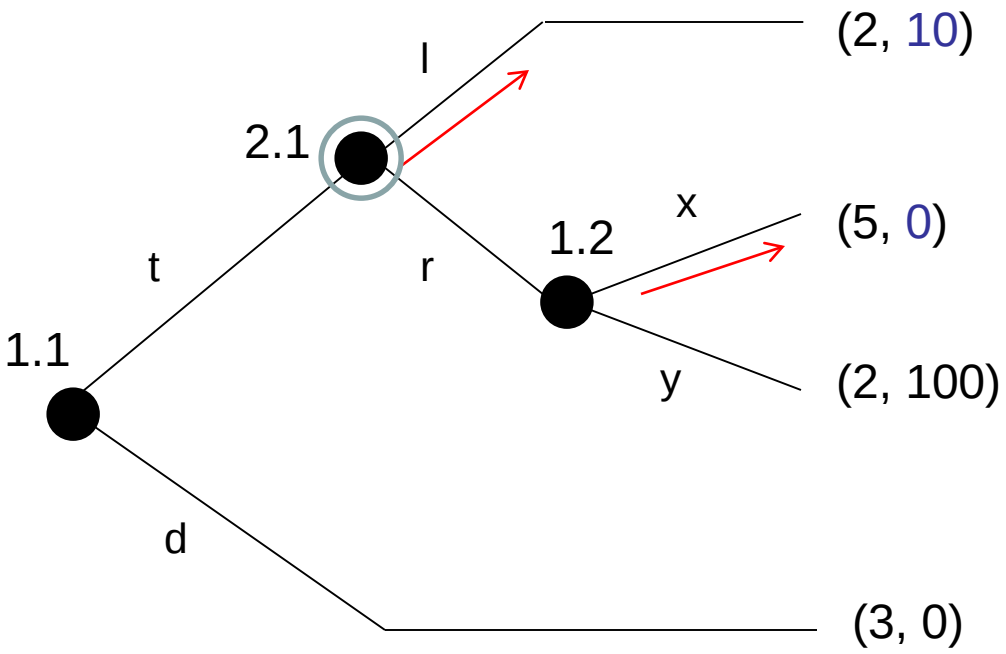
What happens if we attempt forward induction?

Player 2 might play r at 2.1 because he thinks he could get 100.
But if he anticipates 1's move, he will know that 1 will never play y at 1.2.
Backward induction means everyone anticipates the next move and avoids such problems.

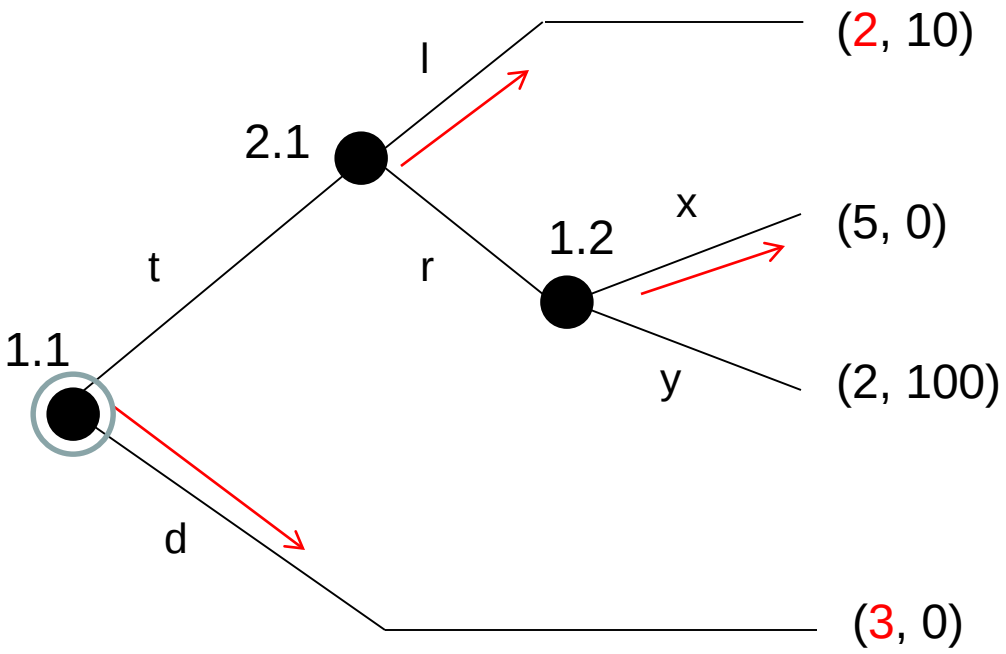
Extensive Form Games



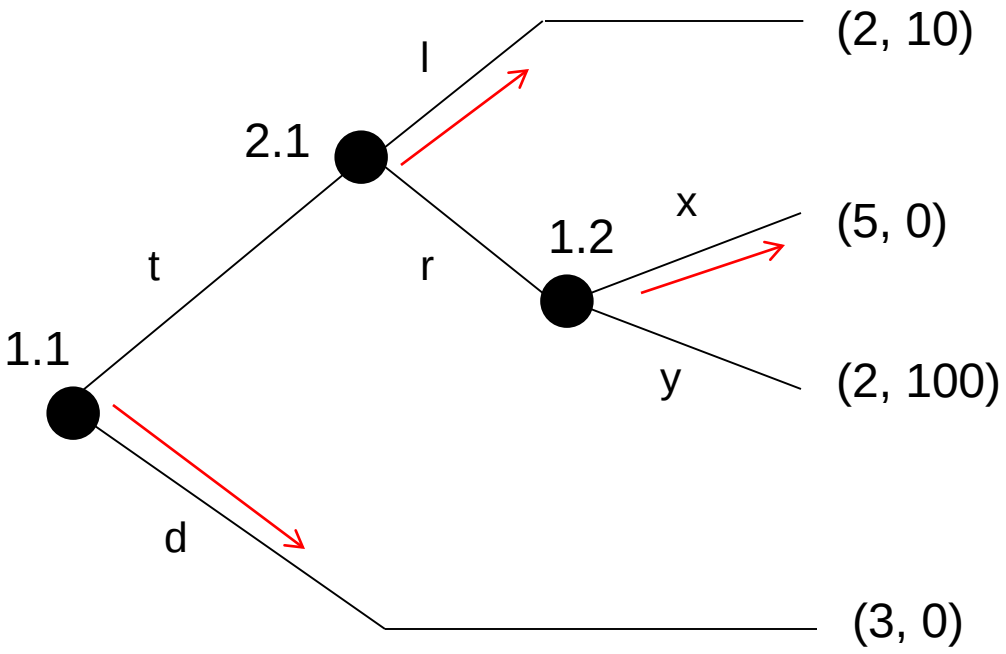
Extensive Form Games



Extensive Form Games



Extensive Form Games



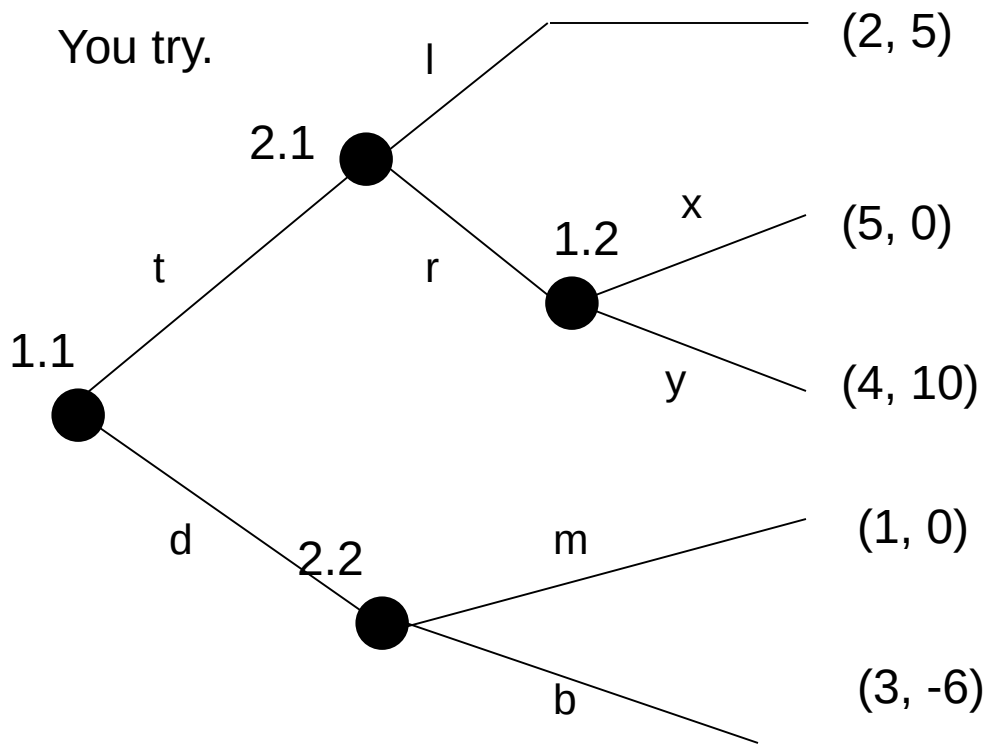
SPE = {(d,x); l}

Note: you must write the full strategy for each player … (d,x) for player 1, even though we never get to x.

Drawing the arrows can be sufficient for showing your work.

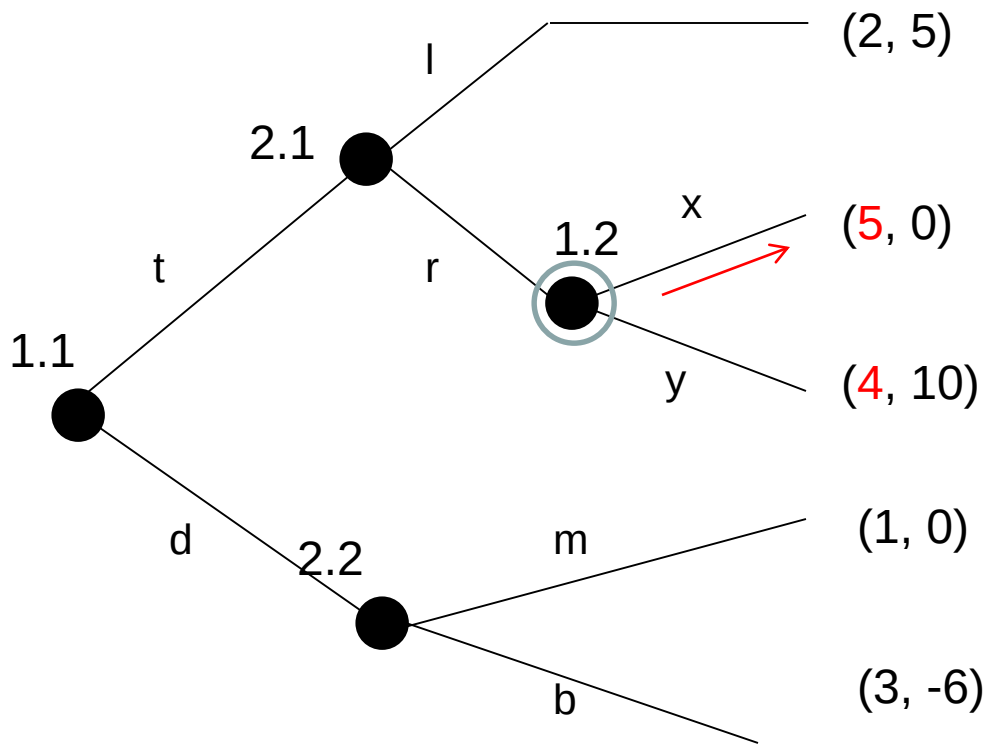
Extensive Form Games

4. Another Example



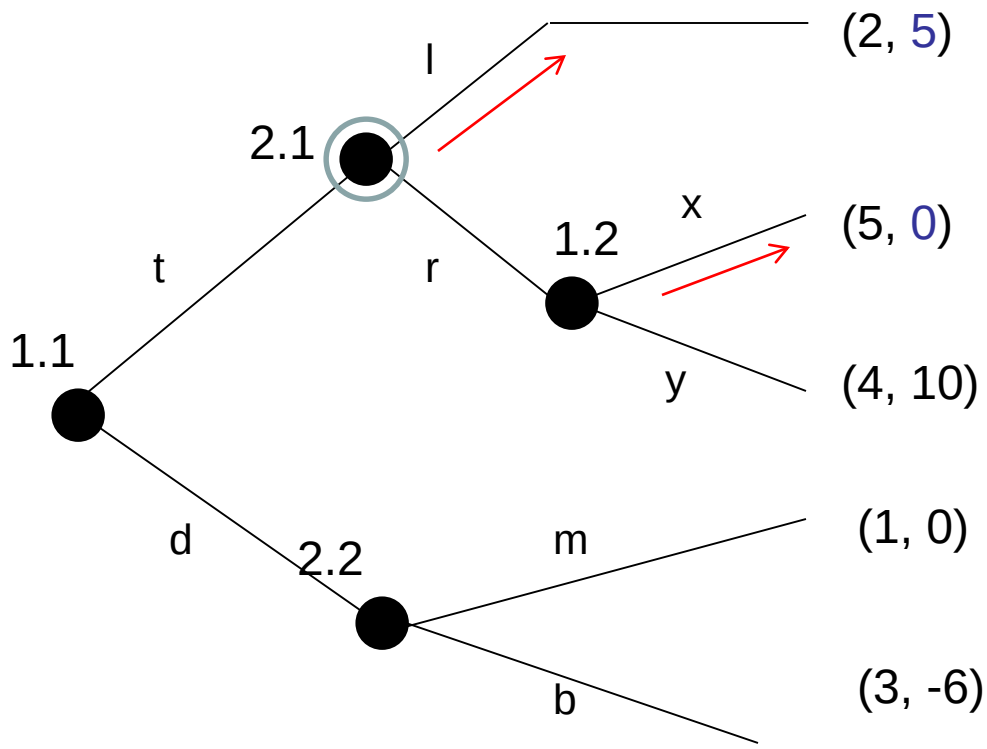
Extensive Form Games

4. Another Example



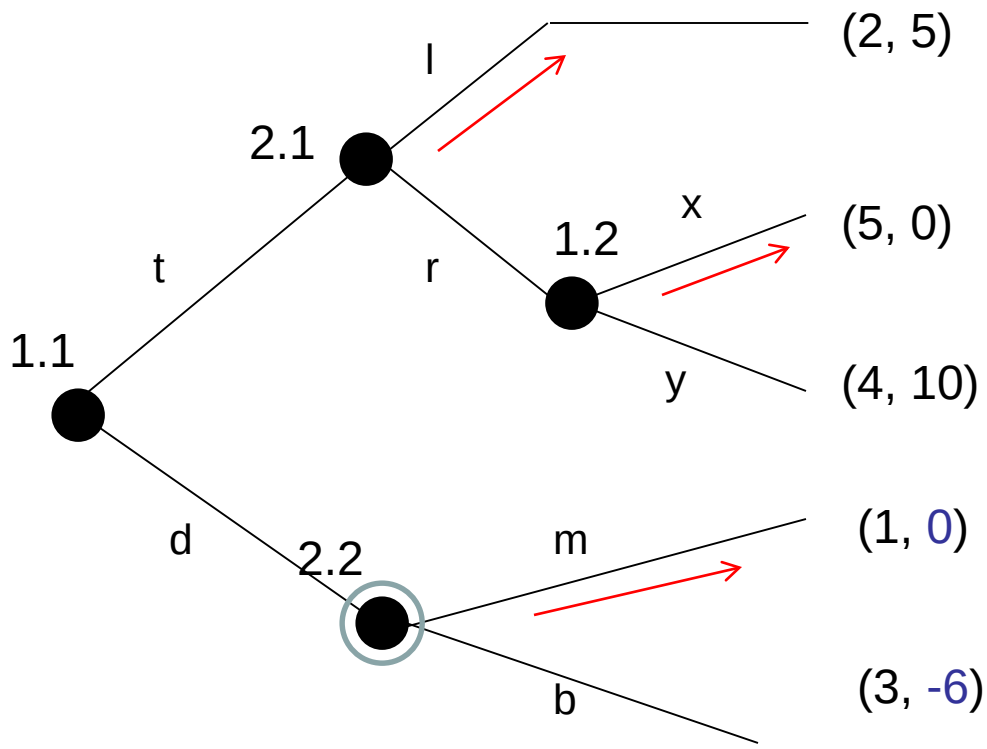
Extensive Form Games

4. Another Example



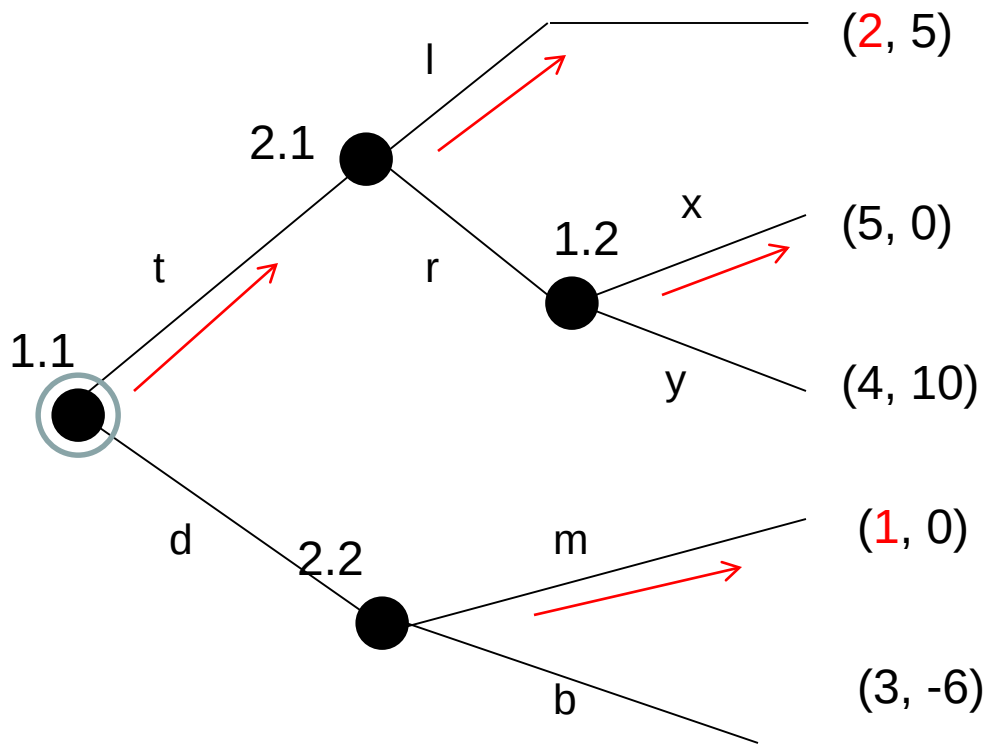
Extensive Form Games

4. Another Example



Extensive Form Games

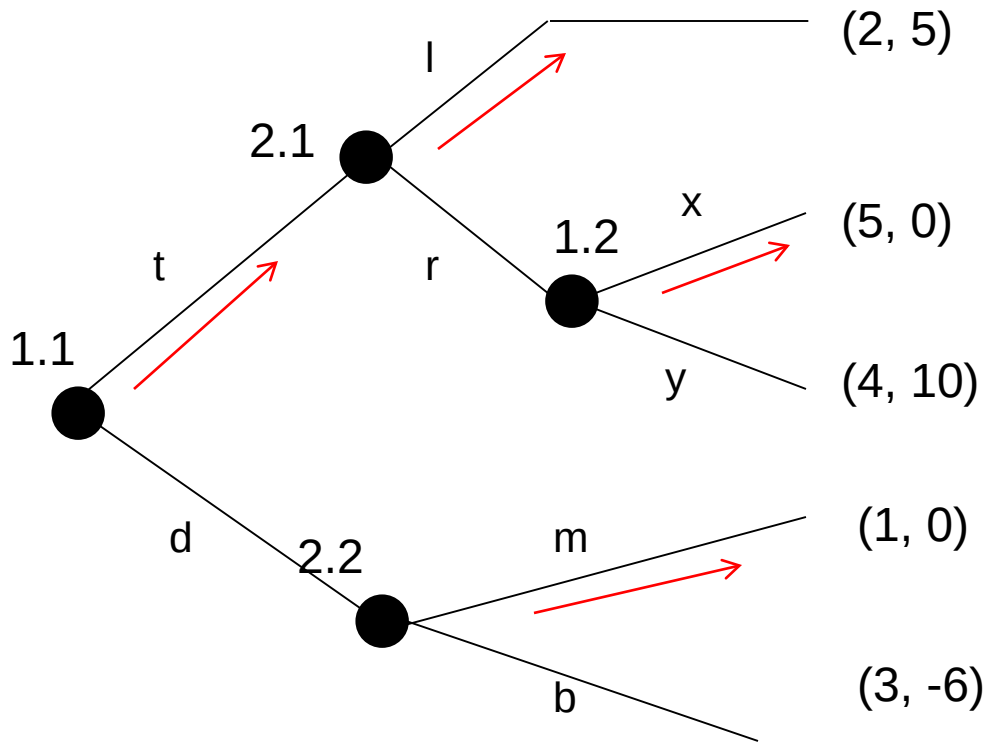
4. Another Example



Extensive Form Games

4. Another Example

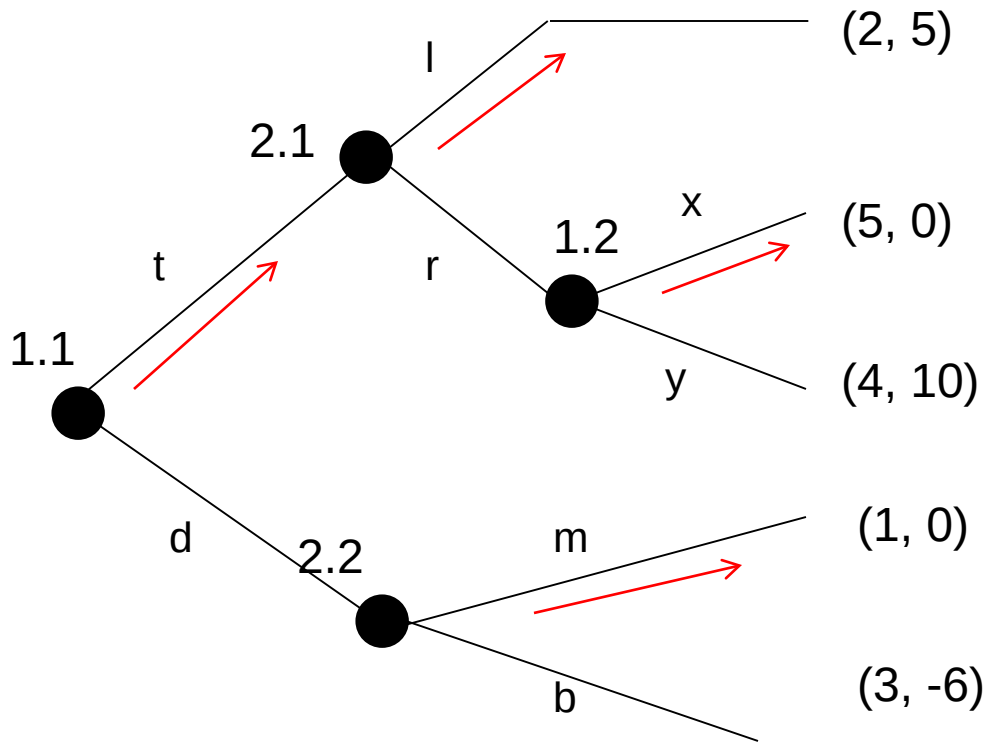
SPE = ?



Extensive Form Games

4. Another Example

SPE = {(t,x);(l.m)}



Notice: If player 1, wasn't going to play x at 1.2 (he plays y instead), player 2 would play r at 2.1, and player 1 would play t at 1.1. That's why we have to write down player 1's commitment to x in the equilibrium. If 1 wasn't committed to x, we would get a different outcome.

Extensive Form Games

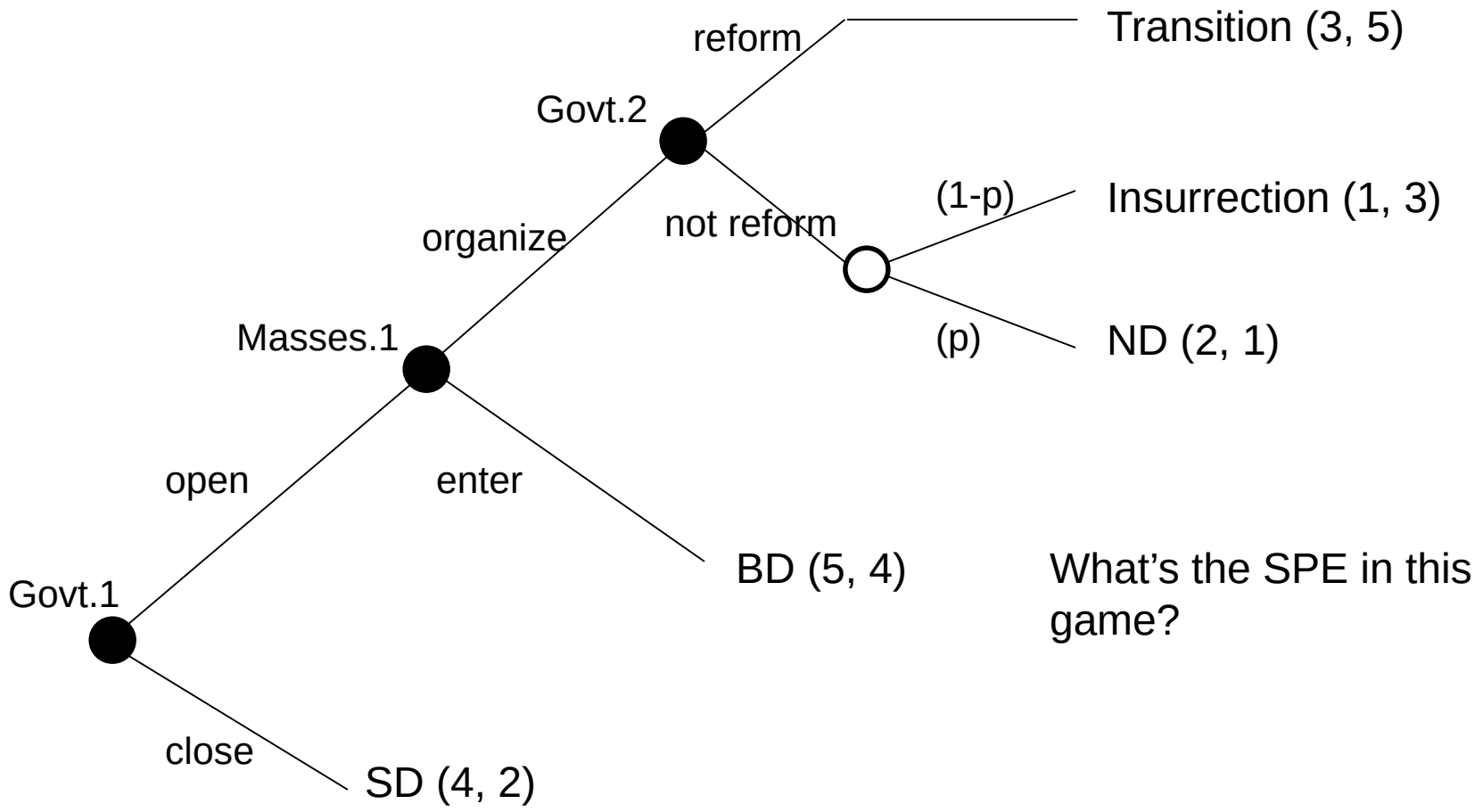
B. The Transition to Democracy

1. Story – think Tiananmen Square in China.

	Preferences	
	Govt	Masses
BD– broad dictatorship (greater liberties, freer markets)	1st	1st
SD– strong dictatorship (remaining hardline)	2nd	2nd
ND– narrow dictatorship (repression, martial law, curfews)	3rd	3rd

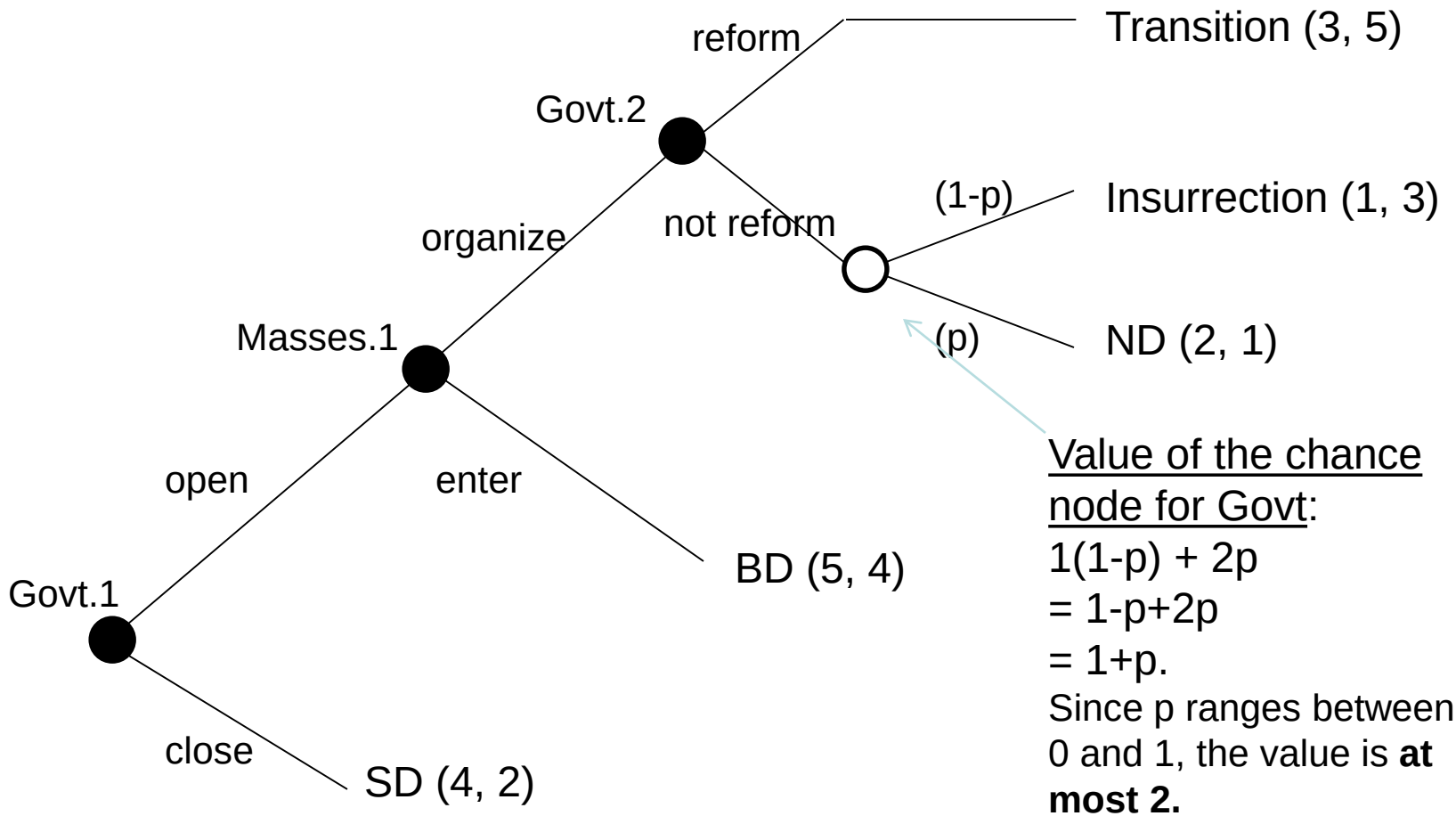
The interesting part of this story is both the reformist government and the masses prefer BD to SD, but they won't get their mutually desired outcome.

Extensive Form Games



What's the SPE in this game?

Extensive Form Games



Value of the chance node for Govt:

1(1-p) + 2p

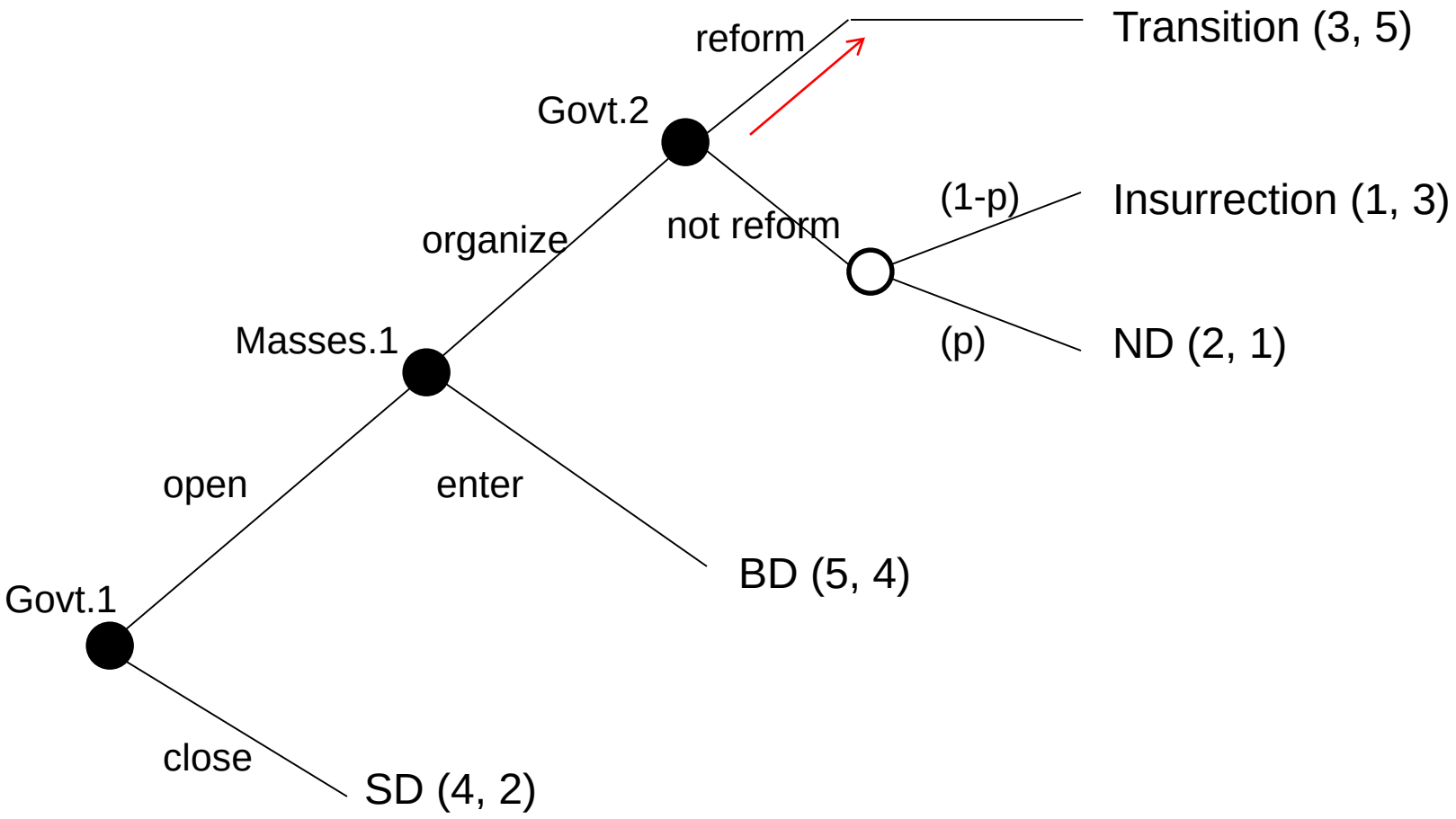
= 1-p+2p

= 1+p.

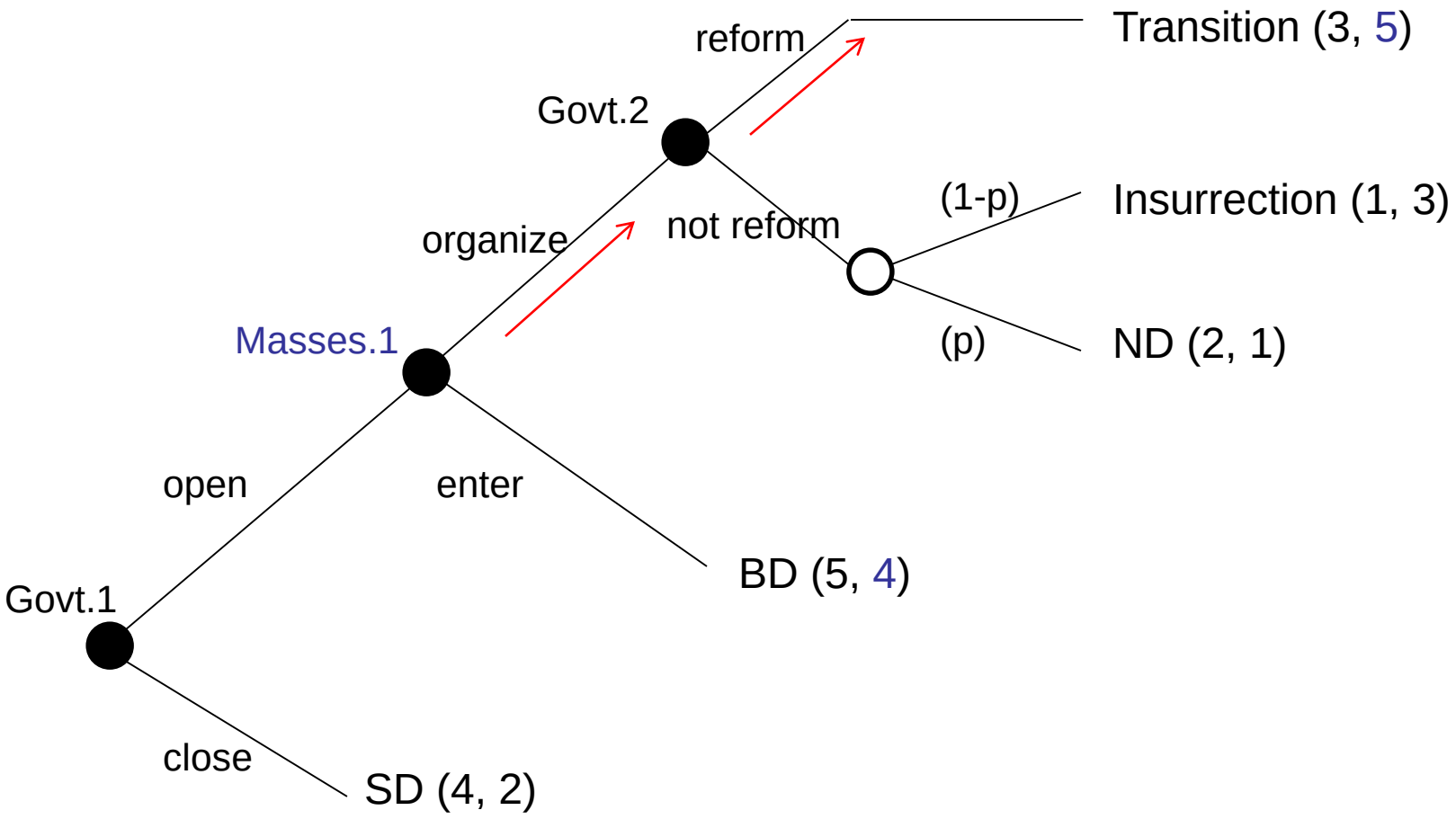
Since p ranges between 0 and 1, the value is **at most 2.**

Therefore, govt prefers reform to not reform.

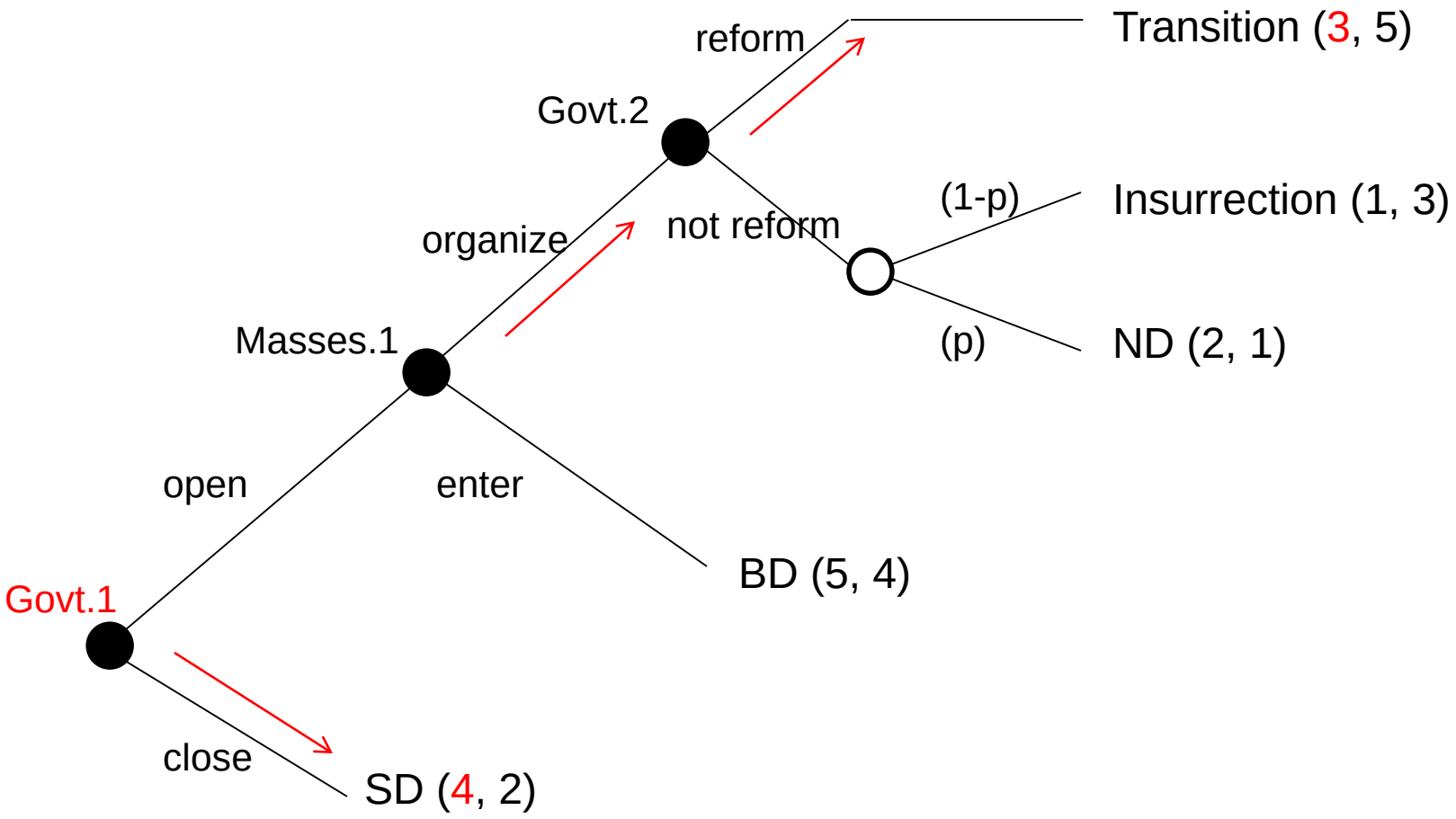
Extensive Form Games



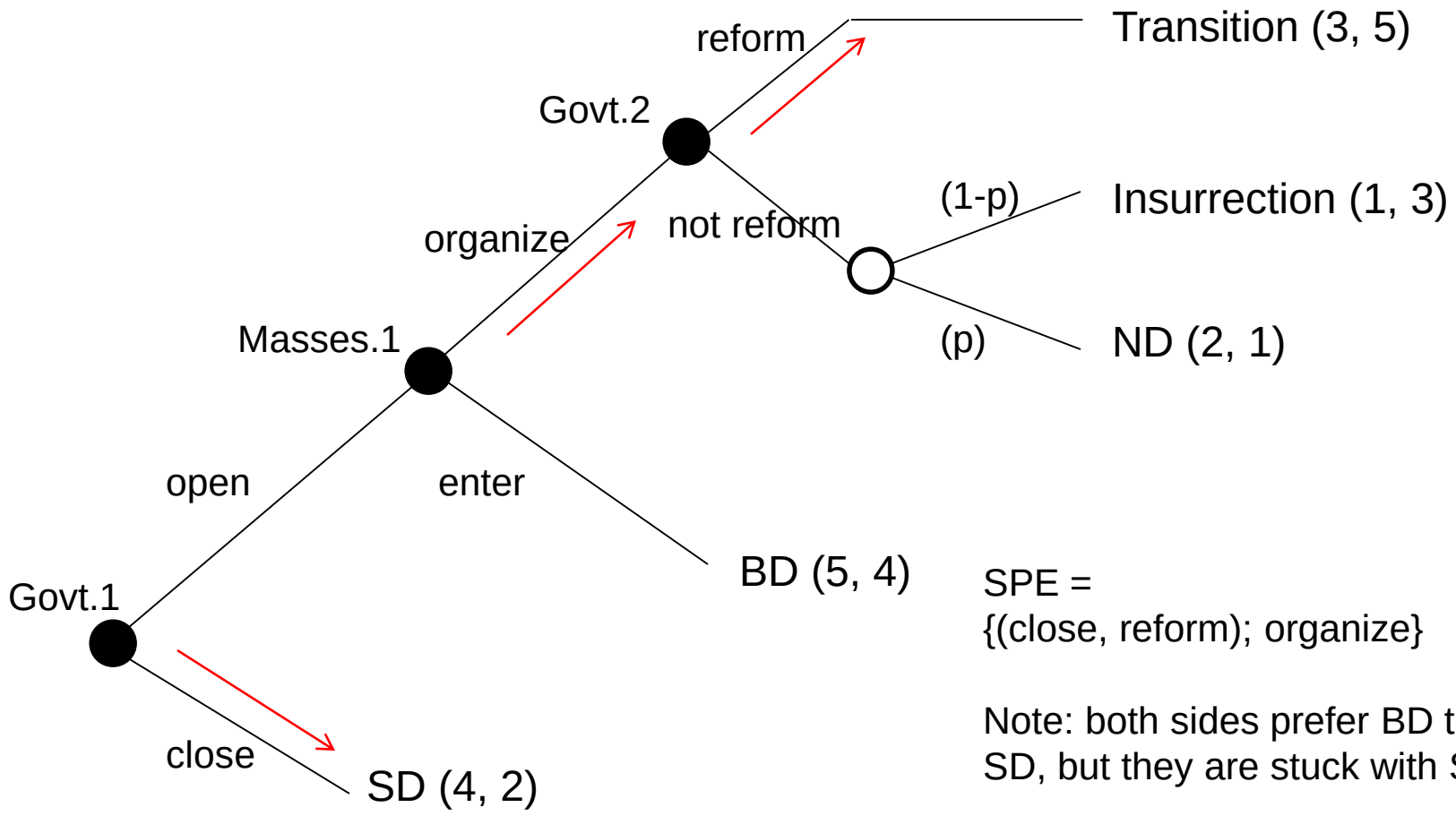
Extensive Form Games



Extensive Form Games



Extensive Form Games



SPE =
{(close, reform); organize}

Note: both sides prefer BD to SD, but they are stuck with SD.

Extensive Form Games

2. Other payoffs would lead to different outcomes

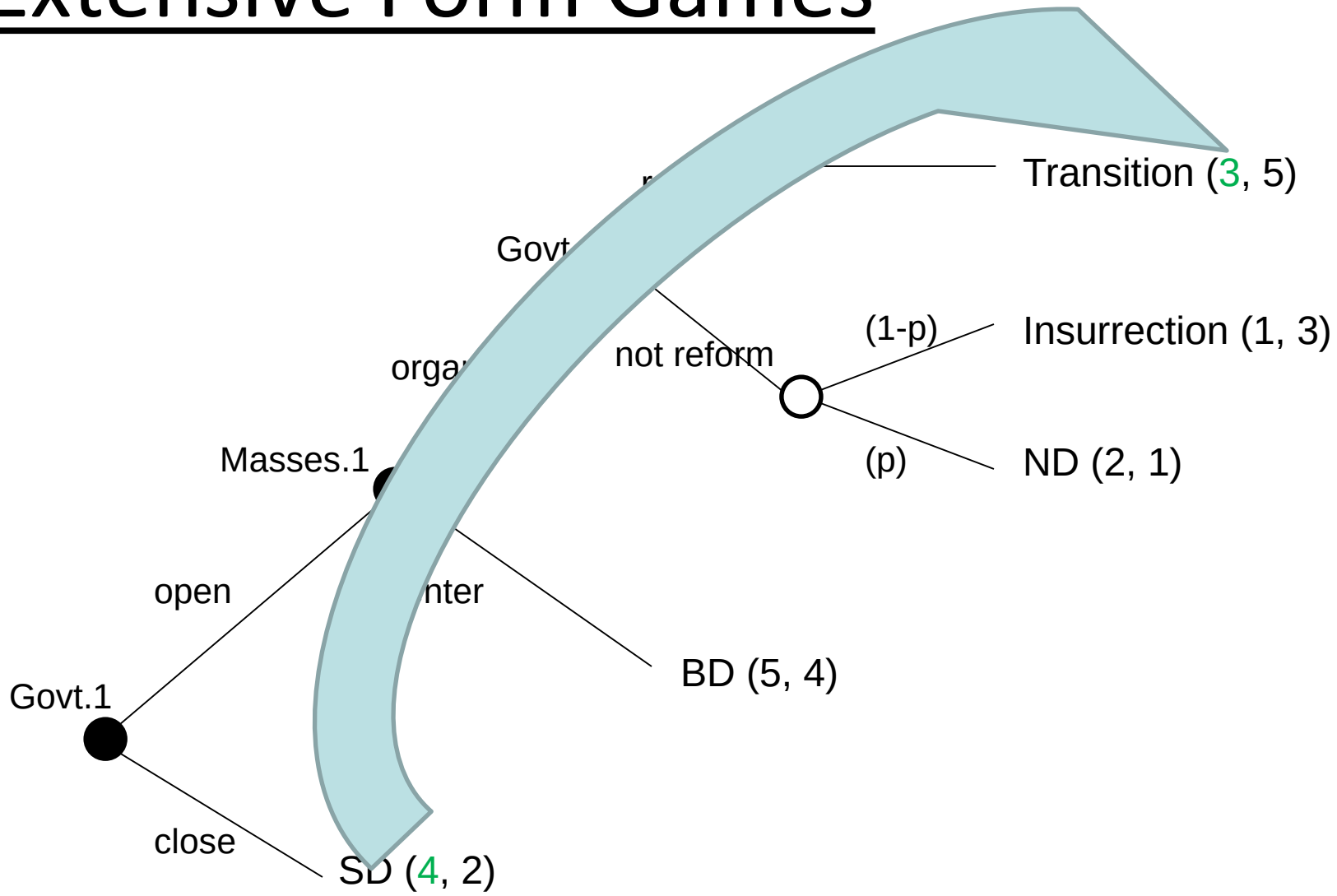
Govt	Masses
BD	transition
SD	BD
transition	insurrection
ND	SD
insurrection	ND

Extensive Form Games

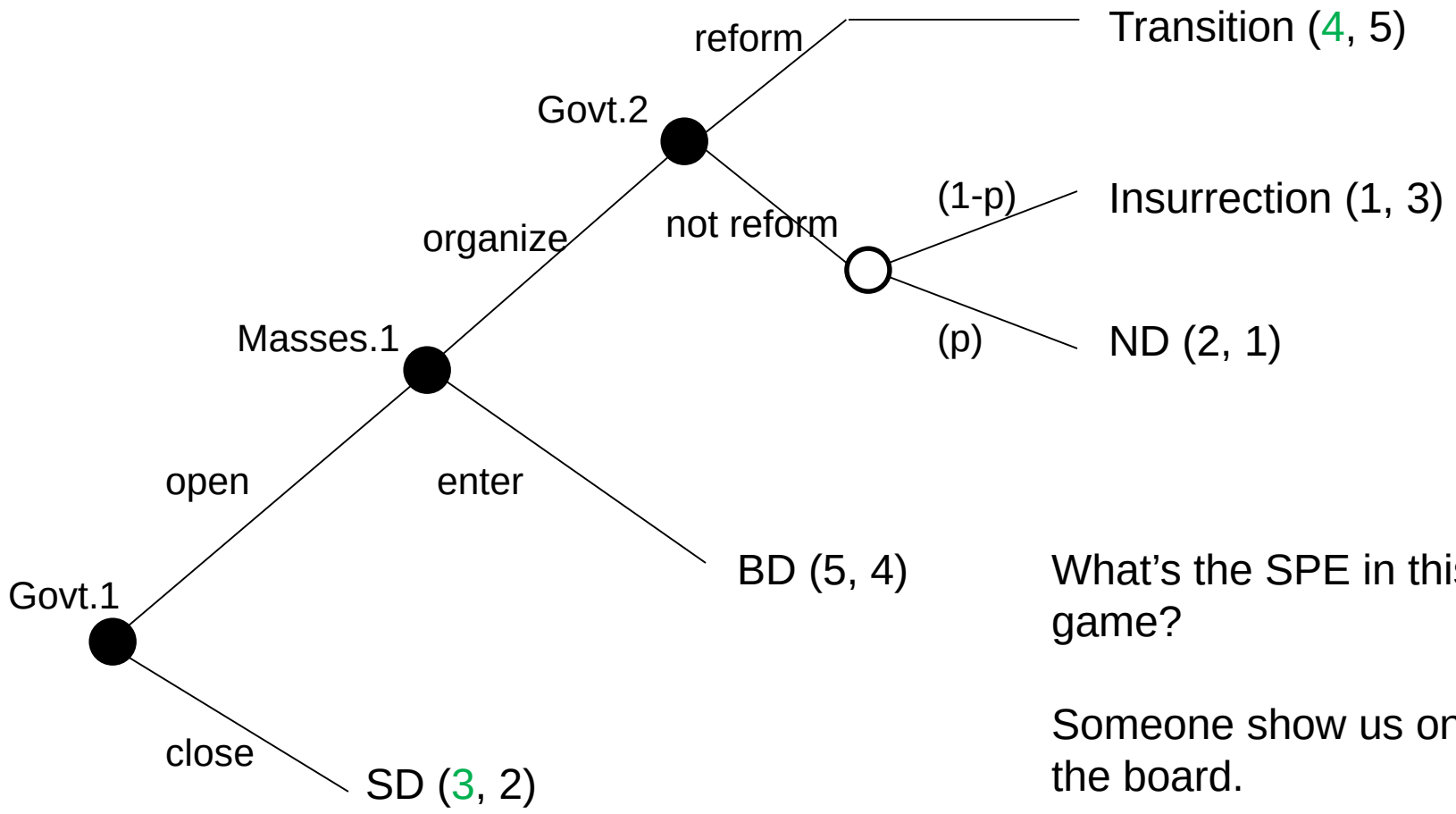
2. Other payoffs would lead to different outcomes

Govt	Masses
BD	transition
transition	BD
SD	insurrection
ND	SD
insurrection	ND

Extensive Form Games



Extensive Form Games



What's the SPE in this game?

Someone show us on the board.