Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	yz, xz	yz	xy	
xzy	xy, xz	xzy	xz		xy, zy	zy
yxz	yz, xz	xz	yxz	yx, yz		yx
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy		zx	zxy	zx, zy
zyx		zy	yx	yx, zx	zx, zy	zyx

1. Assume PAR is **weaky Paretian** and derive social preferences

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	yz, xz	yz	xy	
xzy	xy, xz	xzy	xz		xy, zy	zy
yxz	yz, xz	xz	yxz	yx, yz		yx
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy		zx	zxy	zx, zy
zyx		zy	yx	yx, zx	zx, zy	zyx

2. Where the social preference is unspecified, assume each case (i.e., xPy, yPx, or xIy) then derive further implications (e.g., for (xyz, yzx))

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	yz, xz	xy, yz	xy	
xzy	xy, xz	xzy	xz		xy, zy	zy
yxz	yz, xz	xz	yxz	yx, yz		yx
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy		zx	zxy	zx, zy
zyx		zy	yx	yx, zx	zx, zy	zyx

2. **Case 1**: For (xyz, yzx) assume social preference is xPy

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	xyz	xy, yz	xy	xy
xzy	xy, xz	xzy	xy, xz	xy	xy, zy	xy, zy
yxz	yz, xz	xz	yxz	yx, yz		yx
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy	xy	xy, zx	zxy	zxy
zyx		zy	yx	yx, zx	zx, zy	zyx

3. Assume the PAR is **IIA**. Then for all profiles (xy, yx) $\Rightarrow$ xPy

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	xyz	xyz	xy	xy
xzy	xy, xz	xzy	xy, xz	xy	xy, zy	xy, zy
yxz	yz, xz	xz	yxz	yx, yz		yx
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy	xy	xy, zx	zxy	zxy
zyx		zy	yx	yx, zx	zx, zy	zyx

4. Assume the PAR is **transitive**, so xPy and yPz $\Rightarrow$ xPz

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	xyz	xyz	xy, xz	xy, xz
xzy	xy, xz	xzy	xy, xz	xy, xz	xzy	xzy
yxz	yz, xz	xz	yxz	yxz	xz	yxz
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy	xy	xy, zx	zxy	zxy
zyx		zy	yx	yx, zx	zx, zy	zyx

5. Use **IIA** again: (xz, zx) $\Rightarrow$ xPz

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	xyz	xyz	xy, xz	xy, xz
xzy	xy, xz	xzy	xy, xz	xy, xz	xzy	xzy
yxz	yz, xz	xz	yxz	yxz	xz	yxz
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy	xy	zxy	zxy	zxy
zyx		zy	yx	yx, zx	zx, zy	zyx

6. Use **transitivity** again: zPx and xPy $\Rightarrow$ zPy

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	xyz	xyz	xy, xz	xy, xz
xzy	xzy	xzy	xzy	xzy	xzy	xzy
yxz	yz, xz	xz	yxz	yxz	xz	yxz
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy, zy	xy, zy	xy, zy	zxy	zxy	zxy
zyx	zy	zy	zy, yx	zyx	zx, zy	zyx

7. Use **IIA** again: (zy, yz) $\Rightarrow$ zPy

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	xyz	xyz	xy, xz	xy, xz
xzy	xzy	xzy	xzy	xzy	xzy	xzy
yxz	yz, xz	xz	yxz	yxz	xz	yxz
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy, zy	xy, zy	xy, zy	zxy	zxy	zxy
zyx	zy	zy	zyx	zyx	zx, zy	zyx

8. Use **transitivity** again: zPy and yPx $\Rightarrow$ zPx

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	xyz	xyz	xy, xz	xy, xz
xzy	xzy	xzy	xzy	xzy	xzy	xzy
yxz	yz, xz	xz	yxz	yxz	xz	yxz
yzx	yz, zx	zx	yzx	yzx	zx	yx, zx
zxy	zxy	zxy	zxy	zxy	zxy	zxy
zyx	zx, zy	zx, zy	zyx	zyx	zx, zy	zyx

9. Use **IIA** again: (zx, xz) $\Rightarrow$ zPx

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	xyz	xyz	xy, xz	xy, xz
xzy	xzy	xzy	xzy	xzy	xzy	xzy
yxz	yxz	yx, xz	yxz	yxz	yx, xz	yxz
yzx	yzx	yx, zx	yzx	yzx	yx, zx	yx, zx
zxy	zxy	zxy	zxy	zxy	zxy	zxy
zyx	zyx	zyx	zyx	zyx	zyx	zyx

11. Use **IIA** again: (yx, xy) $\Rightarrow$ yPx

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	xyz	xyz	xy, xz	xy, xz
xzy	xzy	xzy	xzy	xzy	xzy	xzy
yxz	yxz	yxz	yxz	yxz	yxz	yxz
yzx	yzx	yx, zx	yzx	yzx	yx, zx	yx, zx
zxy	zxy	zxy	zxy	zxy	zxy	zxy
zyx	zyx	zyx	zyx	zyx	zyx	zyx

12. Use **transitivity** again: yPx and xPz $\Rightarrow$ yPz

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xyz	xyz	xyz	xyz	xyz
xzy	xzy	xzy	xzy	xzy	xzy	xzy
yxz	yxz	yxz	yxz	yxz	yxz	yxz
yzx	yzx	yzx	yzx	yzx	yzx	yzx
zxy	zxy	zxy	zxy	zxy	zxy	zxy
zyx	zyx	zyx	zyx	zyx	zyx	zyx

13. Use **IIA** again: (yz, zy) $\Rightarrow$ yPz

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xyz	xyz	xyz	xyz	xyz
xzy	xzy	xzy	xzy	xzy	xzy	xzy
yxz	yxz	yxz	yxz	yxz	yxz	yxz
yzx	yzx	yzx	yzx	yzx	yzx	yzx
zxy	zxy	zxy	zxy	zxy	zxy	zxy
zyx	zyx	zyx	zyx	zyx	zyx	zyx

14. Person 1 is a **dictator**!

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	yz, xz	yx, yz	xy	
xzy	xy, xz	xzy	xz		xy, zy	zy
yxz	yz, xz	xz	yxz	yx, yz		yx
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy		zx	zxy	zx, zy
zyx		zy	yx	yx, zx	zx, zy	zyx

15. **Case 2**: For (xyz, yzx) assume social preference is yPx

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xzy	yxz	yzx	zxy	zyx
xzy	xyz	xzy	yxz	yzx	zxy	zyx
yxz	xyz	xzy	yxz	yzx	zxy	zyx
yzx	xyz	xzy	yxz	yzx	zxy	zyx
zxy	xyz	xzy	yxz	yzx	zxy	zyx
zyx	xyz	xzy	yxz	yzx	zxy	zyx

16. **Case 2**: Repeated application of **IIA** and **transitivity** will imply Person 2 is a **dictator**

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	yz, xz	yz	xy	
xzy	xy, xz	xzy	xz		xy, zy	zy
yxz	yz, xz	xz	yxz	yx, yz		yx
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy		zx	zxy	zx, zy
zyx		zy	yx	yx, zx	zx, zy	zyx

17. **Case 3**: For (xyz, yzx) assume social preference is xIy

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	yz, xz	yz, xly	xy	
xzy	xy, xz	xzy	xz		xy, zy	zy
yxz	yz, xz	xz	yxz	yx, yz		yx
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy		zx	zxy	zx, zy
zyx		zy	yx	yx, zx	zx, zy	zyx

17. **Case 3**: For (xyz, yzx) assume social preference is xly

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	yz, xz	yz,xly,xz	xy	
xzy	xy, xz	xzy	xz		xy, zy	zy
yxz	yz, xz	xz	yxz	yx, yz		yx
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy		zx	zxy	zx, zy
zyx		zy	yx	yx, zx	zx, zy	zyx

18. **Transitivity:** xly and yPz $\Rightarrow$ xPz

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	yz, xz	yz,xly,xz	xy	
xzy	xy, xz	xzy	xz		xy, zy	xz, zy
yxz	yz, xz	xz	yxz	yx, yz		yx
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy		zx	zxy	zx, zy
zyx		zy	yx	yx, zx	zx, zy	zyx

19. **IIA:** (xz, zx) $\Rightarrow$ xPz

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	yz, xz	yz,xly,xz	xy	
xzy	xy, xz	xzy	xz		xy, zy	xzy
yxz	yz, xz	xz	yxz	yx, yz		yx
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy		zx	zxy	zx, zy
zyx		zy	yx	yx, zx	zx, zy	zyx

20. **Transitivity:** xPz and zPy $\Rightarrow$ xPy

Proof for special case: 2 people, 3 alternatives

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	xyz	xy, xz	yz, xz	yz,xly,xz	xy	
xzy	xy, xz	xzy	xz		xy, zy	xzy
yxz	yz, xz	xz	yxz	yx, yz		yx
yzx	yz		yx, yz	yzx	zx	yx, zx
zxy	xy	xy, zy		zx	zxy	zx, zy
zyx		zy	yx	yx, zx	zx, zy	zyx

21. But this contradicts **IIA** with respect to xy!

(xyz, yzx) $\Rightarrow$ xIy

(xzy, zyx) $\Rightarrow$ xPy