Introduction to Applied Bayesian Modeling for the Social Sciences

ICPSR 2012

Thursday, June 21, 2012
Due Monday, June 25, 2012

On your homework, please indicate your *name*, *home institution*, *field of study*, and your *e-mail address*.

- Prove that the gamma distribution,

$$p(\theta|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}, \quad \theta, \alpha, \beta > 0 \tag{1}$$

  is the conjugate prior distribution for θ in a Poisson probability mass function,

$$p(y_i|\theta) = \frac{e^{-\theta}\theta^{y_i}}{y_i!} \tag{2}$$

  that is, calculate a form for the posterior distribution of θ and show that it is also gamma distributed.

- Now use the Gamma-Poisson conjugate specification to analyze data on the number of presidential appointments from 1960 to 2000. The data are in the file called `hw.1.bayes.dta` on the Z-drive.

- The posterior distribution for θ is gamma(δ_1, δ_2) according to some parameters δ_1 and δ_2 that you derived above which of course depend on your choice of the parameters for the gamma prior. You should model θ using two sets of priors. One which specifies a great deal of certainty regarding your best guess as to the value of θ and one that represents ignorance regarding this value.

- Generate a large number of values from this distribution in `R`, say $10,000$ or so, using the command:
  `posterior.sample <- rgamma(10000,d1,d2)`

- Summarize the posteriors with quantities of interest such as means, medians, and variances. Also supply plots of the density of the posterior distributions.