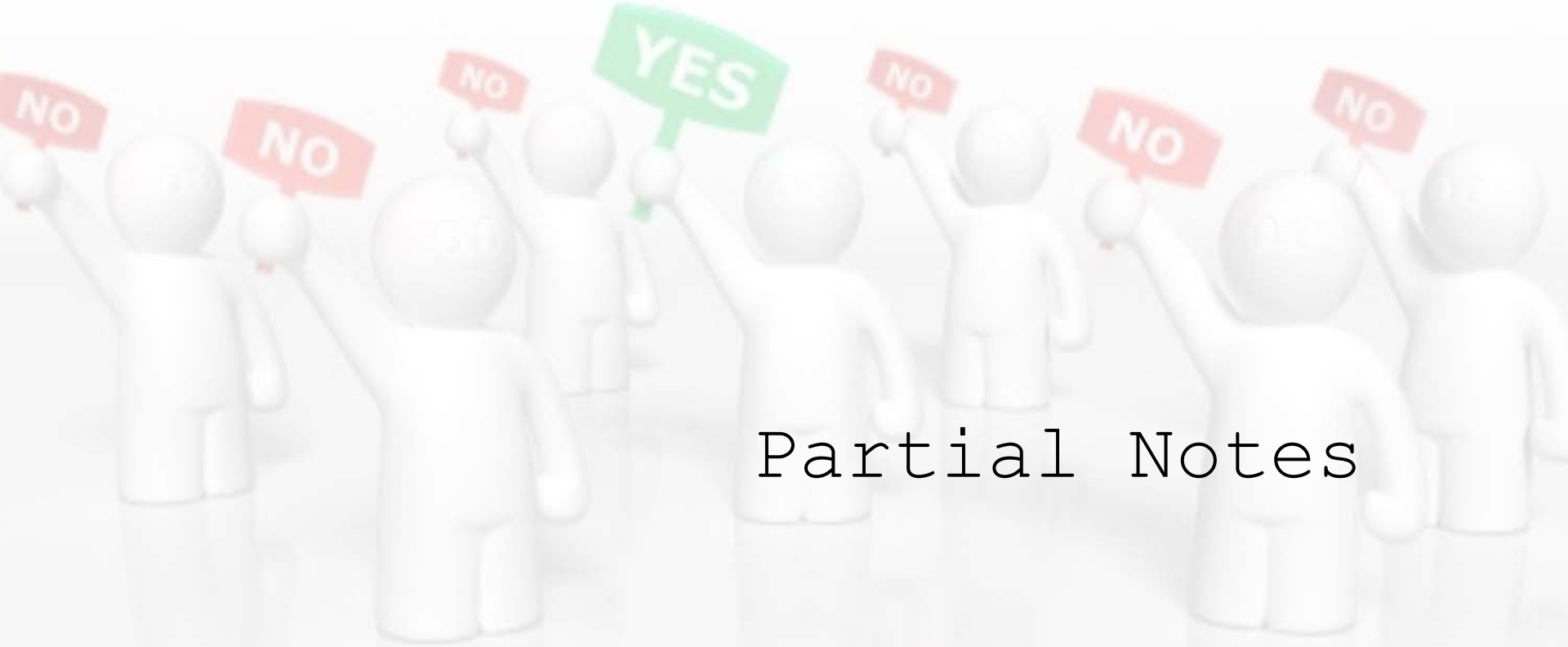

Partial Notes

ARROW'S THEOREM

Arrow's Theorem

C. Intuition behind the theorem

1. Ramsey Center has some money to buy new equipment.

   a. Students propose following expenditures:

      Elliptical machines (e), camping gear (c), squat cages (s), more rock climbing (r).

      Center wants to rank spending priorities based on student preferences.

Individual Preferences

1	2	3	4	5
e	e	r	c	s
c	r	s	r	c
s	s	c	e	e
r	c	e	s	r

Unrestricted Domain (U) – says that any order of individual preferences are allowed.

Here's one for five individuals.

Arrow's Theorem

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Here's another.

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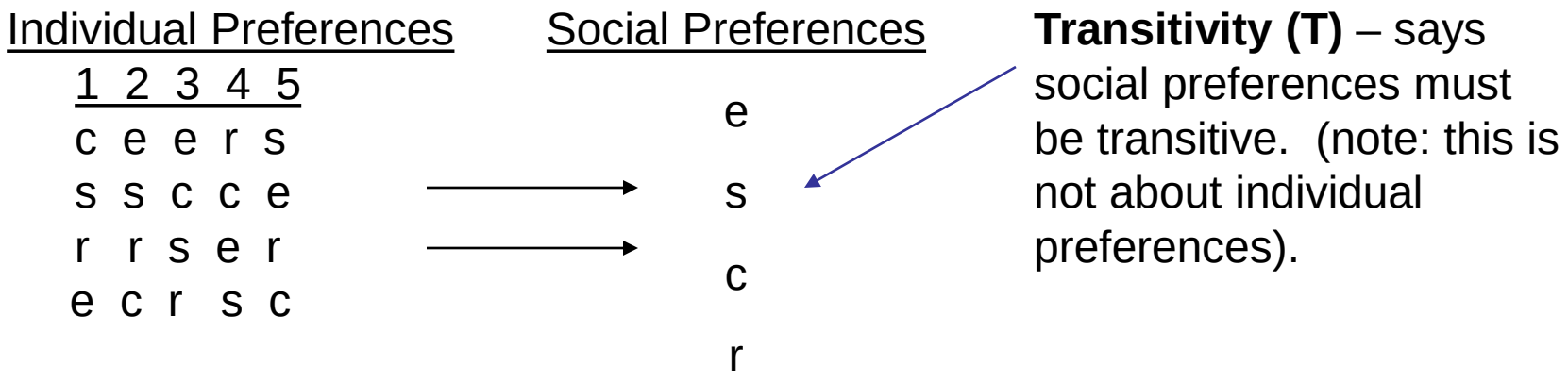

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Individual Preferences					Social Preferences
1	2	3	4	5	
c	e	e	c	s	e
s	s	c	s	e	→ s
r	r	s	e	r	→ c
e	c	r	r	c	r

Pareto (P) –if everyone in a society prefers x to y, then society should prefer x to y .

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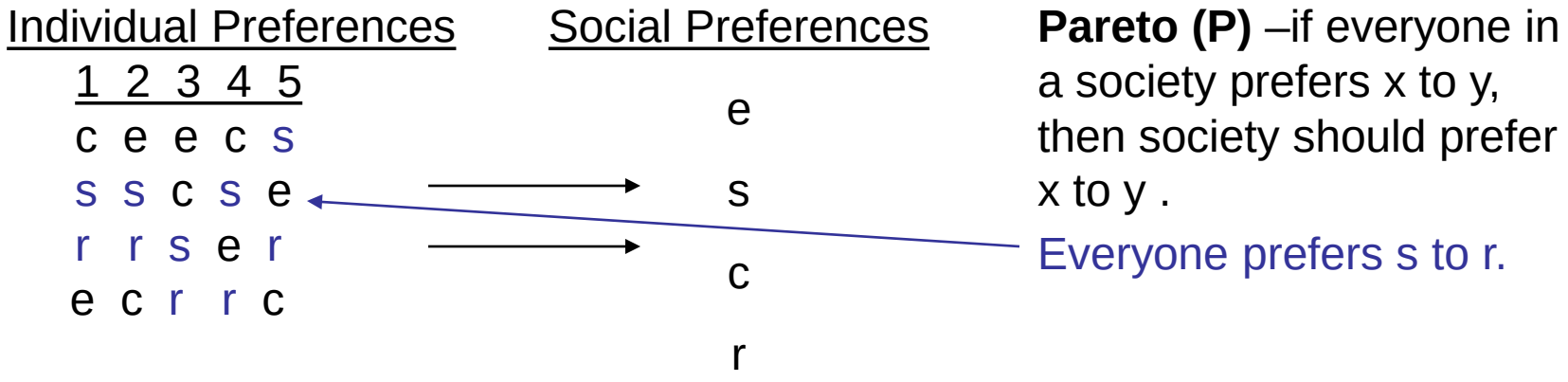

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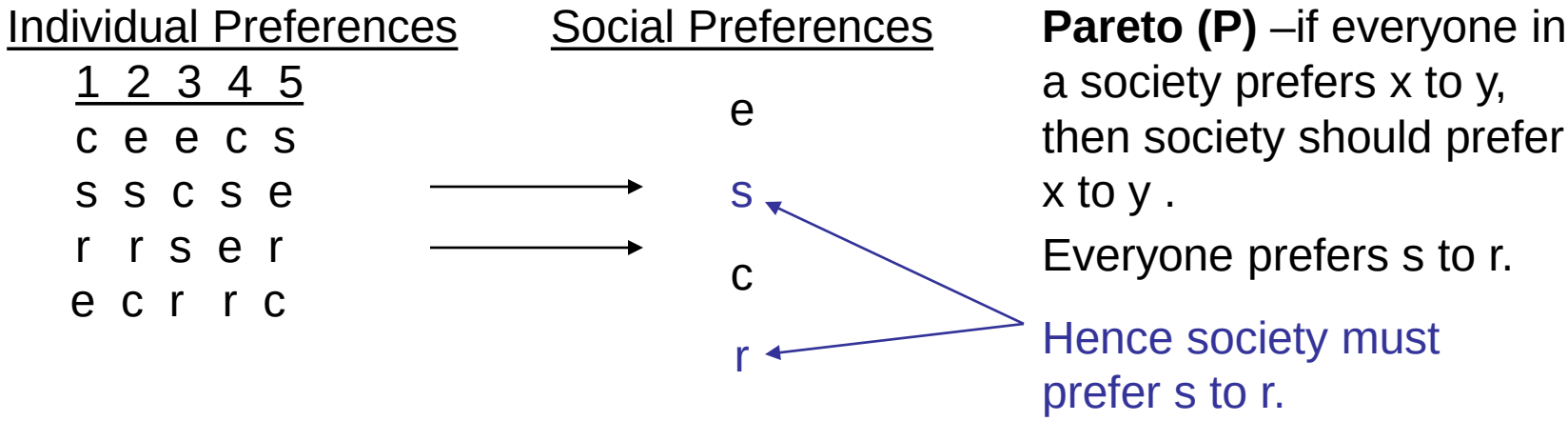

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1	2	3	4	5	
c	e	e	c	s	e
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IIA – for any pair of alternatives (such as e and s),

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1	2	3	4	5	
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s	s	c	s	e	s
r	r	s	e	r	c
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IIA – for any pair of alternatives (such as e and s), the social ranking of that pair of alternatives (e and s)

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1	2	3	4	5	
c	e	e	c	s	e
s	s	c	s	e	→ s
r	r	s	e	r	→ c
e	c	r	r	c	r

IIA – for any pair of alternatives (such as e and s), the social ranking of that pair of alternatives (e and s) should be independent of the individual rankings of other pairs (such as c and r).

Arrow's Theorem

C. Intuition behind the theorem

1. Ramsey Center has some money to buy new equipment.
   a. Students propose following expenditures:

Hence, if one or more individuals switched r and c (but left their ranking of e and s unchanged), the social ranking of e and s should be unchanged.

Individual Preferences					Social Preferences
1	2	3	4	5	
c	e	e	c	s	e
s	s	c	s	e	→ s
r	r	s	e	r	→
e	c	r	r	c	c
					r

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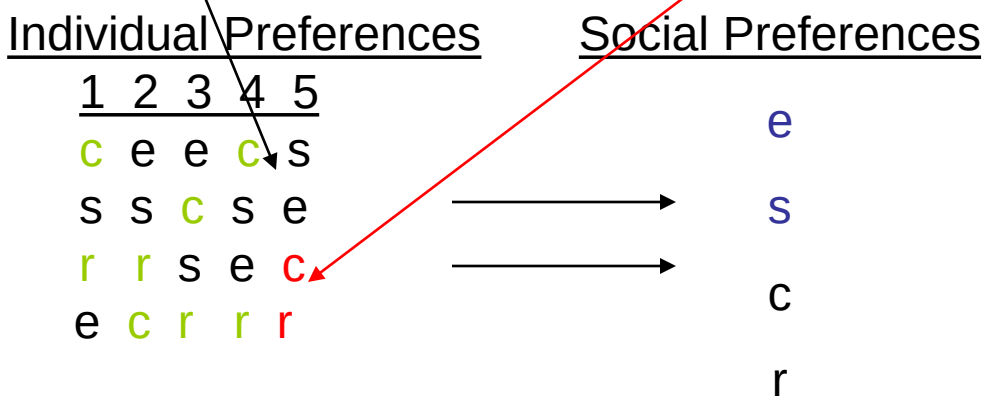

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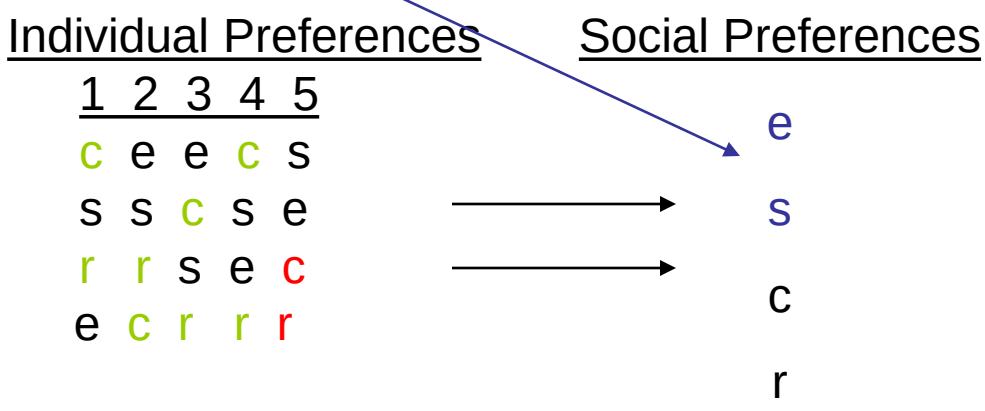

IIA – for any pair of alternatives (such as e and s), the social ranking of that pair of alternatives (e and s) should be independent of the individual rankings of other pairs (such as c and r).

Arrow's Theorem

C. Intuition behind the theorem

1. Ramsey Center has some money to buy new equipment.
   a. Students propose following expenditures:

Hence, if one or more individuals switched r and c (but left their ranking of e and s unchanged), the social ranking of e and s should be unchanged. This is true for any "exalted" pair (e and s in this case) and any number of switches of "irrelevant pairs" (c & r, c & e, c & s, etc…).

Individual Preferences					Social Preferences
1	2	3	4	5	
c	e	e	c	s	e
s	s	c	s	e	s
r	r	s	e	c	c
e	c	r	r	r	r

IIA – for any pair of alternatives (such as e and s), the social ranking of that pair of alternatives (e and s) should be independent of the individual rankings of other pairs (such as c and r).

IIA

Critical Note

IIA is not the same as independence from eliminated alternatives (IEA)!

Hodge and Kilma get them confused. You must use the version of IIA stated here (or in your readings from Sen) to get Arrow's Theorem correct.

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Individual Preferences		Social Preferences
1 2 3 4 5		e
c e e c s		
s s c s e	→	s
r r s e r	→	c
e c r r c		
		r

Nondictatorship (ND) – there cannot be one individual who determines the group's ranking regardless of the other individuals' rankings.

Arrow's Theorem

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1. Ramsey Center has some money to buy new equipment.
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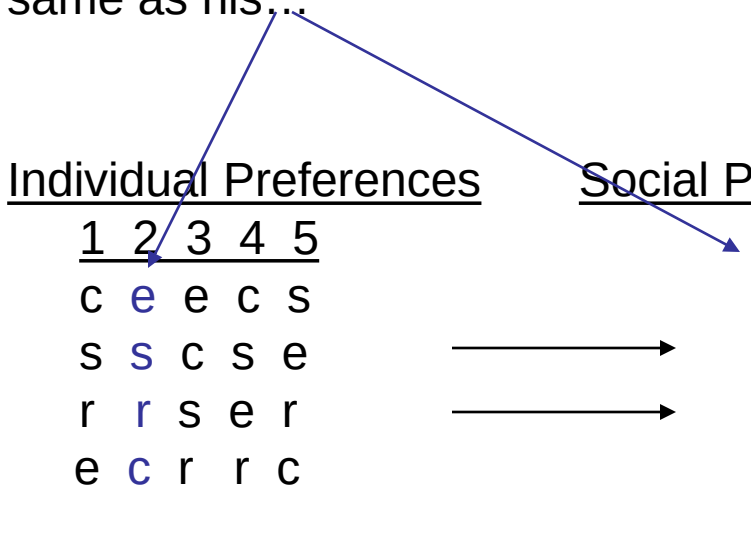
Individual 2 would be a dictator if society's preferences were always the same as his…



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Individual Preferences		Social Preferences
1 2 3 4 5		e
c e c c c		
s s r s r	→	s
r r s e e	→	r
e c e r s		
		c

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Arrow's Theorem

D. Pedagogical Proof

- see other power point.