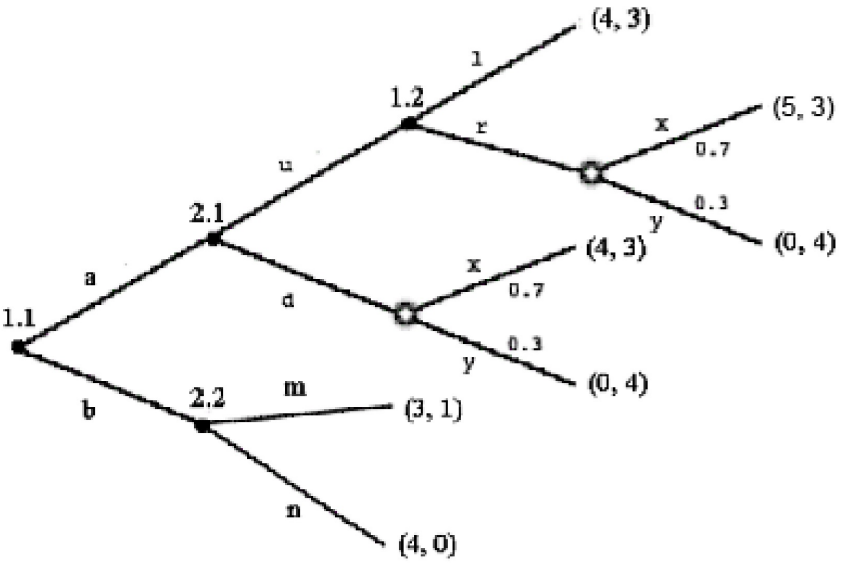
POLS 8000
Introduction to Rational Choice
Dougherty

Homework 2: Game Theory
(due: in class, Feb 14)

Directions: The following questions should help you understand the material in the first part of the course. Please write your answers *neatly* on a separate sheet of paper. You will probably have to re-write your answers before you turn them in. Furthermore:

1) You are welcome to ask background and clarification questions from other students in the class, but you must attempt to work through as much of the homework as possible on your own, and then work with other students only when you are stuck or want to check your answers.
2) You need to write up your own answers, using your own words and explanations. If you turn in the exact same answer as another student, I will consider it academic dishonesty.
3) Every answer needs an explanation which includes how you derived the solution. Please justify your response, include requisite math, and **SHOW YOUR WORK**.
4) Late homeworks will be reduced one letter grade for every working day they are late (folks want to see the answers).
5) Each sub-question is worth 11 pts, with 1 bonus pt for turning the homework in on time.

1. Consider the following extensive form game. In this game there are two players, named 1 and 2. Player 1 has choices between *a* and *b* at her first decision node (1.1) and between *l* and *r* at her second decision node (1.2) – note: (1.2) means player 1, node 2. Player 2 has choices between *u* and *d* at his first decision node (2.1) and between *m* and *n* at his second decision node (2.2). If player 1 chooses *a* and player 2 chooses *d*, then nature will chose *x* with a probability of 0.7 and *y* with a probability of 0.3. If player 1chooses *a* then *r* and player 2 chooses *u*, then nature will chose a different x with a with a probability of 0.7 and a different *y* with a probability of 0.3. Payoffs for player 1 and 2 are marked.



a. What is the expected payoff to player 1 from the play of (a, d)?

b. What is the expected payoff to player 2 from the play of (a, d)?

c. What is the subgame perfect equilibrium of this game? [hint: your answers to the previous two questions are useful here. You also might start with similar answers for the other decision node(s).]

2. Consider the following two person, normal form game

		2			
		a	b	c	d
1	A	3, 3	0, 0	3, 0	3, 1
	B	4, 6	3, 3	5, −1	−5, 0
	C	0, 1	3, 6	4, 1	0, −1

a. What are the *actions* available to player 1? What are the *strategies* available to player 1? How do they differ both in this case and theoretically?

b. What is / are the strictly dominant strategy equilibria? (note: you need not consider mixed strategies. Strictly dominant means "$>$" not "$\geq$"). [*Hint*: Row's payoffs are on the left. Column's payoffs are on the right. To strike a row, look only at row's payoffs. To strike a column, look only at column's payoffs].

c. What is / are the Nash equilibria? (note: you need not consider mixed strategies)

3. Consider the following two person zero sum game.

		Defense		
Offense		Standard	Nickel	Stacked
	Pass	0	−2	5
	Run	4	−2	4

a. Convert this zero-sum game with one payoff in each box to a normal form game with two payoffs in each box.

b. What is / are the strictly dominant strategy equilibria? (note: you need not consider mixed strategies. Use the hints from question 2].

c. What is / are the Nash equilibria? (note: you need not consider mixed strategies)

4. EXTRA CREDIT (10 points): Imagine there is a committee with 3 members. On February 14 there will be a vote about whether to pass proposition x or keep the status quo q. Two of the members (type 1) prefer x to q and one of the members (type 2) prefers q to x. Every individual knows every other individual's preferences and they also know that the game will be a one-shot vote over the two alternatives only. Each player could take one of the following actions: show up and vote x (vx), show up and vote q (vq), or not show up (dnv). They benefit 10 utiles if the alternative they prefer passes, 0 otherwise. However, they will also incur the cost of 2 utiles if they vote (regardless of whether their favorite alternative passes or not).

Voting will proceed using simple majority rule. That is, the proposition x passes if yeas exceed the nays. Otherwise, the proposition fails.

Write out the payoffs for each outcome and determine all the pure strategy Nash equilibria in the game. For each equilibrium tell me whether the proposition would pass or fail. Remember to show your work. [hint: the discussion of 3 player games in the Dixit, Riley, Skeath book may be of some use here. I will not give partial credit on the extra credit, so it's all or nothing. Good luck.].