POLS 8000
Introduction to Rational Choice
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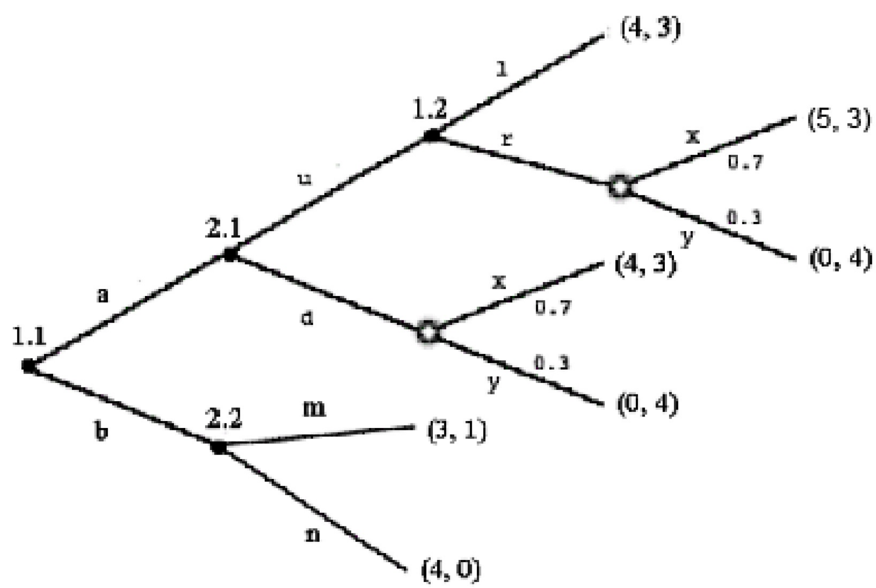
**Homework 2: Game Theory**
(answers)

1. Consider the following extensive form game. In this game there are two players, named 1 and 2. Player 1 has choices between *a* and *b* at her first decision node (1.1) and between *l* and *r* at her second decision node (1.2) – note: (1.2) means player 1, node 2. Player 2 has choices between *u* and *d* at his first decision node (2.1) and between *m* and *n* at his second decision node (2.2). If player 1 chooses *a* and player 2 chooses *d*, then nature will chose *x* with a probability of 0.7 and *y* with a probability of 0.3. If player 1chooses *a* then *r* and player 2 chooses *u*, then nature will chose a different x with a with a probability of 0.7 and a different *y* with a probability of 0.3. Payoffs for player 1 and 2 are marked.




a. What is the expected payoff to player 1 from the play of (a, d)?

**At this chance node, player 1 will receive a payoff of 4 with a probability of .7 and a payoff of 0 with a probability of .3. Hence:**

**$EU_1 = 4 (.7) + 0 (.3) = 2.8$**

b. What is the expected payoff to player 2 from the play of (a, d)?

**At this chance node, player 2 will receive a payoff of 3 with a probability of .7 and a payoff of 4 with a probability of .3. Hence:**

**$EU_2 = 3 (.7) + 4 (.3) = 3.3$**

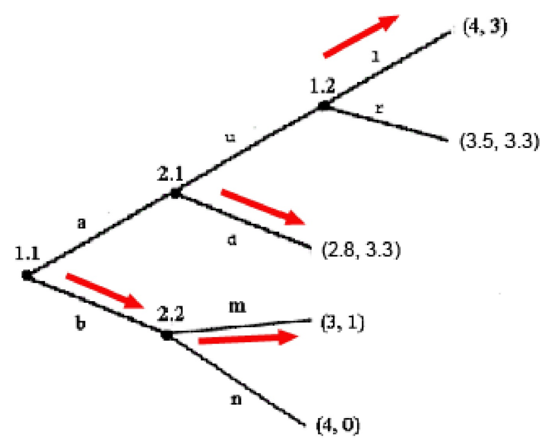
c. What is the sequential equilibrium of this game? [hint: your answers to the previous two questions are useful here. You also might start with similar answers for the other decision node.]

**At the second chance node, player 1 will receive a payoff of 5 with a probability of .7 and a payoff of 0 with a probability of .3. Hence:**

**$EU_1 = 5\ (.7) + 0\ (.3) = 3.5$**

**At the second chance node, player 2 will receive $EU_2 = 3.3$, for the same reasons as above (question 1b).**

**This allows us to simplify the game into the following tree.**




**We then solve by backward induction as shown by the arrows in the tree. At node 1.2, player 1 chooses l because 4 > 3.5. At node 2.1, player 2 chooses d because he knows that player 1 will play l at 1.2 giving him 3 and she prefers payoff 3.3 to payoff 3. At node 2.2, player 2 chooses m over n because 1 > 0. Hence, at 1.1 player 1 chooses b because a implies d and 2.8 while b implies m and 3. Since 3 > 2.8, player 1 plays b.**

**S.E. = {(b, l); (d, m)}.**

2. Consider the following two person, normal form game

| 1 \ 2 | a | b | c | d |
|---|---|---|---|---|
| A | 3, 3 | 0, 0 | 3, 0 | 3, 1 |
| B | 4, 6 | 3, 3 | 5, −1 | −5, 0 |
| C | 0, 1 | 3, 6 | 4, −1 | 0, −1 |

a. What are the *actions* available to player 1? What are the *strategies* available to player 1? How do they differ both in this case and theoretically?

**Actions are the choices that an individual has at a given decision node. A strategy is a complete list of possible actions for a particular player throughout the game (at all of a player's decision nodes). In other words, directions about what actions to take at every decision node. Since there is only one decision node for each player in this case (i.e. one decision node), the actions are the same as the strategies. For example, A, B, and C are both player 1's actions and his strategies.**

b. What is / are the strictly dominant strategy equilibria? (note: you need not consider mixed strategies. Strictly dominant means ">" not "≥"). [*Hint*: Row's payoffs are on the left. Column's payoffs are on the right. To strike a row, look only at row's payoffs. To strike a column, look only at column's payoffs].

| | | 2 | | | |
|---|---|---|---|---|---|
| | | a | b | c | d |
| 1 | A | 3, 3 | 0, 0 | 3, 0 | 3, 1 |
| | B | 4, 6 | 3, 3 | 5, −1 | −5, 0 |
| | C | 0, 1 | 3, 6 | 4, 1 | 0, −1 |

**It doesn't matter where we start so let's start by striking columns. Remember when you are striking columns, only look at column's payoffs (right of the comma). Strike d, because a dominates d (since 3 > 1; 6 > 0; and 1 > -1). The strike is indicated by the gray area above. We "cannot" strike c because a only weakly dominates c (since 1 in cell (C,a) is equal to 1 in cell (C,c). No more column strikes are possible at this time.**

| | | 2 | | | |
|---|---|---|---|---|---|
| | | a | b | c | d |
| 1 | A | 3, 3 | 0, 0 | 3, 0 | 3, 1 |
| | B | 4, 6 | 3, 3 | 5, −1 | −5, 0 |
| | C | 0, 1 | 3, 6 | 4, 1 | 0, −1 |

**Now try to strike rows. Since the remainder of B dominates the remainder of A, we can strike A (since 4 > 3; 3 > 0; and 5 > 3). The strike is indicated by the gray area above. No more row strikes are possible at this time.**

| | | 2 | | | |
|---|---|---|---|---|---|
| | | a | b | c | d |
| 1 | A | 3, 3 | 0, 0 | 3, 0 | 3, 1 |
| | B | 4, 6 | 3, 3 | 5, −1 | −5, 0 |
| | C | 0, 1 | 3, 6 | 4, 1 | 0, −1 |

**Now try to strike columns again. Since the remainder of b dominates the remainder of c, we can strike c (since 3 > -1; 6 > 1). The strike is indicated by diagonal lines above. Since we can make no more column strikes and no more strikes, we are done. SDSE = {B; a}, {B; b}, {C; a}, {C; b}.**

c. What is / are the Nash equilibria? (note: you need not consider mixed strategies)
**The Best Reply Graph is as follows. I highlighted the best reply in each case. Think of these as the circles.**

| | | 2 | | | |
|---|---|---|---|---|---|
| | | a | b | c | d |
| 1 | A | 3, 3 | 0, 0 | 3, 0 | 3, 1 |
| | B | 4, 6 | 3, 3 | 5, −1 | −5, 0 |
| | C | 0, 1 | 3, 6 | 4, 1 | 0, −1 |

**The Nash equilibrium is the one where two payoffs are circled (i.e. both payoffs are highlighted). In this case, N.E. = {B; a}, {C, b}.**

3. Consider the following two person zero sum game.

| Offense | | Defense | | |
|---|---|---|---|---|
| | | Standard | Nickel | Stacked |
| | Pass | 0 | −2 | 5 |
| | Run | 4 | −2 | 4 |

a. Convert this zero-sum game with one payoff in each box to a normal form game with two payoffs in each box.

| Offense | | Defense | | |
|---|---|---|---|---|
| | | Standard | Nickel | Stacked |
| | Pass | 0, 0 | −2, 2 | 5, −5 |
| | Run | 4, −4 | −2, 2 | 4, −4 |

b. What is / are the strictly dominant strategy equilibria? (note: you need not consider mixed strategies. Use the hints from question 2].

| Offense | | Defense | | |
|---|---|---|---|---|
| | | Standard | Nickel | Stacked |
| | Pass | 0, 0 | −2, 2 | 5, −5 |
| | Run | 4, −4 | −2, 2 | 4, −4 |

**It doesn't matter where we start so let's start by striking columns. Remember when you are striking columns, only look at column's payoffs (right of the comma). Strike Standard, because Nickel dominates Standard (since 2 > 0; and 2 > −4). The strike is indicated in dark gray above (standard column).**

**Also strike Stacked, because Nickel dominates Stacked (since 2 > −5; and 2 > −4). The strike is indicated in dark gray above (stacked column).**

**Now strike rows. Remember when you are striking rows, only look at row's payoffs (left of the comma). Since −2 = −2 (in column Nickel), run does not dominate pass and pass does not dominate run.**

**SDSE = {Pass; Nickel}, {Run; Nickel}.**

c. What is / are the Nash equilibria? (note: you need not consider mixed strategies)

**The Best Reply Graph is as follows. I highlighted the best reply in each case. Think of these as the circles.**

| Offense | | Defense | |
|---|---|---|---|
| | Standard | Nickel | Stacked |
| Pass | 0, 0 | −2, 2 | 5, −5 |
| Run | 4, −4 | 2, −2 | 4, −4 |

**The Nash equilibria are the outcomes where two payoffs are circled (i.e. both payoffs are highlighted). In this case, N.E. = {Pass; Nickel}, {Run; Nickel}.**

4. EXTRA CREDIT (10 points): Imagine there is a committee with 3 members. On February 14 there will be a vote about whether to pass proposition $x$ or keep the status quo $q$. Two of the members (type 1) prefer $x$ to $q$ and one of the members (type 2) prefers $q$ to $x$. Every individual knows every other individual's preferences and they also know that the game will be a one-shot vote over the two alternatives only. Each player could take one of the following actions: show up and vote $x$ (vx), show up and vote $q$ (vq), or not show up (dnv). They benefit 10 utiles if their favorite alternative passes, 0 otherwise. However, they will also incur the cost of 2 utiles if they vote (regardless of whether their favorite alternative passes or not).

Voting will proceed using simple majority rule. That is, proposition $x$ passes if the yeas exceed the nays; otherwise $q$ wins.

Write out the payoffs for each outcome and determine all the pure strategy Nash equilibria in the game. For each equilibrium tell me whether the proposition would pass or fail. Remember to show how your work. [hint: the discussion of 3 player games in the text may be of some use here. I will not give partial credit on the extra credit, so it's all or nothing. Good luck.].

**The following actions are available to each player: vote for x (vx), vote for q (vq), and do not vote (dnv). There are several ways to set up a three person game, but perhaps the best way to keep things straight is to follow Dixit, Reiley, and Skeath (your text).**

<u>Without loss of generality, assume</u>:

player 1 prefers x to q;
player 2 prefers x to q; and
player 3 prefers q to x.

<u>Payoffs</u>:

If player 1 or 2 votes,
...and x passes: 8
...and q prevails: -2
If player 1 or 2 doesn't vote,
...and x passes: 10
...and q prevails: 0
If player 3 votes,
...and x passes: -2
...and q prevails: 8
If player 3 doesn't vote,
...and x passes: 0
...and q prevails: 10

which allows us to create the following cells and best reply graphs...

**Player 3**

**Vx**

| | | Player 2 **Vx** | **Vq** | **DNV** |
|---|---|---|---|---|
| Player 1 | **Vx** | 8, 8, -2 | 8, 8, -2 | 8, 10, -2 |
| | **Vq** | 8, 8, -2 | -2, -2, 8 | -2, 0, 8 |
| | **DNV** | 10, 8, -2 | 0, -2, 8 | 10, 10, -2 |

**Vq**

| | | Player 2 **Vx** | **Vq** | **DNV** |
|---|---|---|---|---|
| Player 1 | **Vx** | 8, 8, -2 | -2, -2, 8 | -2, 0, 8 |
| | **Vq** | -2, -2, 8 | -2, -2, 8 | -2, 0, 8 |
| | **DNV** | 0, -2, 8 | 0, -2, 8 | 0, 0, 8 |

**DNV**

| | | Player 2 **Vx** | **Vq** | **DNV** |
|---|---|---|---|---|
| Player 1 | **Vx** | 8, 8, 0 | -2, -2, 10 | 8, 10, 0 |
| | **Vq** | -2, -2, 10 | -2, -2, 10 | -2, 0, 10 |
| | **DNV** | 10, 8, -2 | 0, -2, 10 | 0, 0, 10 |

There is no pure-strategy Nash Equilibrium, because no cell has three circles.