APPLICATION:
GOVERNMENT FORMATION IN PARLIAMENTS
Government Formation


• Trying to understand which parties will form a government in a parliamentary system assuming:
  – A majority of members need to form a government;
  – Multiple parties with different policy preferences;
  – Various recognition rules about the order of proposing a government;
  – Policies that must be accepted by the majority coalition.

• Assumptions (for today):
  – Three parties, two of which are needed to form a majority coalition.
  – Two dimensional policy preferences.
  – Three equidistant parties.
  – Game proceeding in the following order...
Government Formation

Figure 1. Government Formation Process: Probabilistic Selection

Step 1: a party will be randomly chosen as first proposer. 
\( (p_1 + p_2 + p_3 = 1) \).
We will consider other processes for this step shortly.
Step 2: that party will propose a coalition partner and a policy $x$. 
note: $x^{i,j}$ indicates the policy proposed by party $i$ in coalition with party $j$. 
Step 3: the coalition partner will accept or reject the offer based on the policy, $x^{i,j}$, proposed.
Step 4: if the offer is rejected, another proposer is randomly chosen and the process repeats (until an offer is accepted).
Three Recognition Rules

1. Each party equally likely to be a proposer.
2. Fixed order of proposers: party 1, party 2, party 3.
3. Fixed order of proposers: party 1, party 2.
   – Party 3 is omitted perhaps because it is a small party.

• Baron also considers
  – More recognition rules;
  – Parties that are not equidistant;
  – More than three parties.
Three, Equidistant Parties

Figure 2. Equilibrium Indifference Curves and Policy Proposals

Notation:
$t^1$ – party 1’s ideal point.

$x^{12}$ – policy proposal made by party 1 with party 2.

$V_3$ – expected value to party 3 from $x^{12}$. If party 3 cannot expect any more than that, it would be happy with the proposal.
Three, Equidistant Parties

Note: if party 1 made an offer to party 2, it would on the *contract curve* (a line connecting two of the ideal points on an edge of the ideal point set).

Consider otherwise -- a point like $x$ would *not* be proposed because both parties would prefer something on the contract curve to $x$. 
Three, Equidistant Parties

Suppose party 1 is the proposer $j$: $\lambda_{ji}$ – is the proportion of the distance that $x$ appears from $t^j$ to $t^i$.

So if $j$ proposes something with $i$, it will propose:

$$x^{ji}(\lambda_{ji}) = \lambda_{ji}t^j + (1 - \lambda_{ji})t^i, \lambda_{ji} \in [0, 1]$$

The utility of party $i$ from $x^{ji}$ will be

$$U_i(x^{ji}(\lambda_{ji})) = -\lambda_{ji}^2D_{ji}^2 \quad (1)$$

where $D_{ij} = D_{ji}$ is the distance between the ideal points of parties $i$ and $j$.

The utility of party $j$ will be

$$U_j(x^{ji}(\lambda_{ji})) = -(1 - \lambda_{ji})^2D_{ji}^2$$
Party $i$ will support $x^{ji}(\lambda_{ji})$ if and only if it is at least as preferred as the expected outcome from rejecting it and continuing to the next section of a party attempting to form a govt. $U_i(x^{ji}(\lambda_{ji})) \geq V_i$

Which is

$$= -\lambda_{ji}^2 D_{ji}^2 \geq V_i \quad \text{by (1)}$$

$$= \lambda_{ji} \leq \frac{\sqrt{(-V_i)}}{D_{ji}}.$$

So, when the last equation is true, $i$ will accept $j$’s proposal which will be in equilibrium. The exact value of $V_i$ depends on the proposal process.
Equilibria

Case 1: Each party equally likely to be a proposer:
One equilibria is that each party is equally likely to propose a proposal with the other two parties, such that the proposal is 5/9ths of the distance between their ideal point and the ideal point of their coalition partner.

If party 1 was randomly chosen proposer, it would propose these two with equal probability.

\[ V_i = -(4/9)D^2, \ i = 1, 2, 3. \]
Equilibria:

What happens if $t^3$ was pushed out, but remained equidistant to the other parties?

Answer: for a fixed $p$, it would be less likely that party 3 would be included in a government.
Equilibria

Case 2: Fixed order of proposer: party 1, party 2, party 3:
One equilibrium: party 1 makes a proposal to party 2, which party 2 accepts.

Intuition: if party 1 made a deal with party 3, party 3 could reject it knowing that it could make a better deal when it was proposer.
Equilibria

Case 3: Fixed order of proposers: party 1, party 2:
All equilibria have party 1 creating a coalition with party 3. If ideal points are equidistant, the policy is at least as close to party 1 as it is to party 3.

Intuition: party 3 is ready to make a deal, because it will never have an opportunity to make a better deal.

The farther party 3 is away, the more it is willing to make policy concessions.
Concluding thoughts

• Recognition rules matters
  – If each party is equally likely to propose,
    • the equilibrium is a mixed strategy where the expected payoffs to each party are equal (though the observed policy proposed is not).
  – If each party proposes in a fixed order,
    • then in at least one of the equilibria the last party is cut out of the deal.
  – If only the first two parties propose in a fixed order (perhaps because they are bigger than the third),
    • then the first party will always make a deal with the third party.
    • The third party is back in the deal!