INTRODUCTION TO
GAME THEORY
A. Tic-Tac-Toe (loser pays winner $5).
   1. Are there good and bad moves in tic-tac-toe?

   a. yes, at least some times.
   b. def: action – something a player can choose at a particular point in the game.
Game Theory

2. Is there an optimal strategy to play?
   a. **Def:** strategy – a complete set of actions for a player in a game.
   b. Yes, bad moves (or actions) imply bad strategies.

3. If two people play optimal strategies, what’s the outcome?
   a. A tie. …optimal does not mean win.

4. Should we expect two people to play to a tie?
   a. _?_.
   b. **Def:** preference – an individual’s liking of one outcome compared to another (usually expressed in terms of payoffs).
   c. **Def:** rationality – an individual is rational if they chose the strategy that gets them their most preferred outcome (or more preferred outcome, if “most” preferred is not attainable).

   a. Rationality only means something with respect to an individual’s pre-specified valuation of the outcomes.
Aside:

The probability of an event $A$, expressed as $P(A)$, has the following properties:

1. $0 \leq P(A) \leq 1$.
2. $P(A) = 1 - P(\neg A)$.
3. $P(\emptyset) = 0$.
4. For mutual exclusive and exhaustive events $A_1, A_2, \ldots A_n$:
   \[ \sum_{i}^{n} P(A_i) = 1 \]
B. One Player Games (games against nature)
   1. Bill Clinton Decides to Resign over Monica Lewinski.
B. One Player Games (*games against nature*)

1. Bill Clinton Decides to Resign over Monica Lewinski.

Calculate Expected Value:

<table>
<thead>
<tr>
<th>Decision</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resign</td>
<td>-200,000</td>
<td>-200,000</td>
</tr>
<tr>
<td>Not Resign</td>
<td>0.4(-400,000) + 0.6(0)</td>
<td>-160,000</td>
</tr>
</tbody>
</table>

Therefore: don’t resign
Game Theory

B. One Player Games (games against nature)

2. What if we didn’t know the probabilities?

Sensitivity Analysis: Calculate Expected Value

Resign: \(-200,000\)
Not Resign: \(p(-400,000) + (1-p)0\)

Resign if and only if \(-200,000 > p(-400,000)\)
\(\frac{1}{2} < p\)
C. Multiple Player Games

1. Three types
   1. Zero sum games – your gains are my losses.
   2. Cooperative games – games where agreements are binding (skip).
   3. Non-cooperative games – games where coordination of strategy must be done through play alone.
      a) Zero sum games are a special type of non-cooperative game.

2. Ex: Clinton decides whether to resign, Congress responds.
2. Ex: Clinton decides whether to resign, Congress responds.

More definitions

a. Branches – the actions that can be chosen at each decision node (i.e. the lines).
b. Decision node – a point of decision for an actor.
c. Chance node – a point where mother nature moves (i.e. a probabilistic event occurs).
d. Payoff – a player’s valuation for an outcome (also known as utility).
EXTENSIVE FORM
(SEQUENTIAL MOVE) GAMES
Extensive Form Games

A. Backward Induction

1. Start at the back of the game, determine what is rational at each node, then work forward.

2. subgame perfect equilibrium (SPE) – the expected outcome of the game, determined by backward induction.

SPE = {resign; find guilty}

(-200,000, 3)

(-400,000, 2)

(0, 1)

SPE = {resign; find guilty}

… always written first player’s strategy; second player’s strategy; etc.
3. Another Example

2.1 means player 2, node 1
Extensive Form Games

What happens if we attempt forward induction?

Player 2 might play r at 2.1 because he thinks he could get 100.
What happens if we attempt forward induction?

Player 2 might play r at 2.1 because he thinks he could get 100. But if he anticipates 1’s move, he will know that 1 will never play y at 1.2.
What happens if we attempt forward induction?

Player 2 might play r at 2.1 because he thinks he could get 100. But if he anticipates 1’s move, he will know that 1 will never play y at 1.2.

Backward induction means everyone anticipates the next move and avoids such problems.
Extensive Form Games
Extensive Form Games

1.1

2.1

1.2

(3, 0)

(2, 100)

(2, 10)

(5, 0)
Extensive Form Games

Diagram:

- Starting node 1.1, with actions t, r, and d.
- Node 2.1 linked to nodes 1.2 and r.
- Node 1.2 linked to nodes x and y.
- Nodes x and y lead to outcomes (2, 10) and (2, 100), respectively.
- Node r leads to outcomes (5, 0) and (2, 100), respectively.
- Node t leads to outcome (3, 0).

Outcomes:
- (3, 0) for t
- (5, 0) for r
- (2, 10) for x
- (2, 100) for y
- (2, 10) for l
- (2, 100) for d
Extensive Form Games

Note: you must write the full strategy for each player … (d,x) for player 1, even though we never get to x.

Drawing the arrows can be sufficient for showing your work.

SPE = {(d,x); l}
4. Another Example

You try.

1.1 → d
2.1 → t

1.2 → r
2.2 → m

You try.

(2, 5)
(5, 0)
(4, 10)
(1, 0)
(3, -6)
4. Another Example

Extensive Form Games
4. Another Example

```
1.1  
  t   r
  2.1
    l

1.2
  x
  y
  2.2
    m
      b
  (1, 0)
  (3, -6)
  (4, 10)
  (5, 0)
  (2, 5)
```
Extensive Form Games

4. Another Example

```
2.1
 t   l
2.2
 d   m
```

```
1.1
 r   x
1.2
 v
```

```
(2, 5)
(5, 0)
(4, 10)
(1, 0)
(3, -6)
```
Extensive Form Games

4. Another Example

\[ (3, -6) \]
\[ (2, 5) \]
\[ (5, 0) \]
\[ (4, 10) \]
\[ (1, 0) \]
\[ (3, -6) \]
4. Another Example

Extensive Form Games

SPE = ?
4. Another Example

SPE = \{ (t,x); (l,m) \}

Notice: If player 1 wasn’t going to play x at 1.2 (he plays y instead), player 2 would play r at 2.1, and player 1 would play t at 1.1. That’s why we have to write down player 1’s commitment to x in the equilibrium. If 1 wasn’t committed to x, we would get a different outcome.
Extensive Form Games

B. The Transition to Democracy


<table>
<thead>
<tr>
<th></th>
<th>Govt</th>
<th>Masses</th>
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<tbody>
<tr>
<td>BD—</td>
<td>1st</td>
<td>1st</td>
</tr>
<tr>
<td>SD—</td>
<td>2nd</td>
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</tr>
<tr>
<td>ND—</td>
<td>3rd</td>
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BD— broad dictatorship (greater liberties, freer markets)
SD— strong dictatorship (remaining hardline)
ND— narrow dictatorship (repression, martial law, curfews)

The interesting part of this story is both the reformist government and the masses prefer BD to SD, but they won’t get their mutually desired outcome.
Extensive Form Games

Govt.1

Govt.2

Masses.1

Not reform

Transition (3, 5)

Insurrection (1, 3)

ND (2, 1)

BD (5, 4)

What’s the SPE in this game?

Govt.1

open

close

enter

organize

reform

(p)

(1-p)

SD (4, 2)
Extensive Form Games

- **Govt.1**
  - open
  - close
  - SD (4, 2)

- **Masses.1**
  - open
  - enter

- **Govt.2**
  - Organize
  - not reform
  - reform
  - Transition (3, 5)
  - Insurrection (1, 3)
  - ND (2, 1)

The value of the chance node for Govt is:

\[(1-p) + 2p = 1-p + 2p = 1+p.\]

Since \(p\) ranges between 0 and 1, the value is at most 2. Therefore, govt prefers reform to not reform.
Extensive Form Games

Govt.1
- open
- close

Masses.1
- open
- enter

Govt.2
- reform
- not reform
  - (1-p) Transition (3, 5)
  - (p) Insurrection (1, 3)
  - ND (2, 1)

SD (4, 2)
BD (5, 4)
Extensive Form Games

Govt.1
- open
  - Masses.1
    - enter
      - reform
        - Govt.2
          - reform
            - Transition (3, 5)
          - (1-p)
            - Insurrection (1, 3)
        - (p)
          - ND (2, 1)
      - not reform
        - SD (4, 2)
        - BD (5, 4)
Extensive Form Games

Govt.1

Masses.1

Govt.2

reform

not reform

open

enter

close

SD (4, 2)

BD (5, 4)

Insurrection (1, 3)

ND (2, 1)

Transition (3, 5)

(p)

(1-p)
Extensive Form Games

Govt.1

open

Govt.2

reform

not reform

(1-p)

Insurrection (1, 3)

(p)

ND (2, 1)

Transition (3, 5)

Masses.1

enter

organize

SPE = 
{(close, reform); organize}

Govt.1

close

SD (4, 2)

BD (5, 4)

Note: both sides prefer BD to SD, but they are stuck with SD.
# Extensive Form Games

2. Other payoffs would lead to different outcomes

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Extensive Form Games

- SD (4, 2)
- Govt.1 close
- Masses.1 open
- Govt. enter

- ND (2, 1)
- (p)
- (1-p)
- Insurrection (1, 3)
- Transition (3, 5)

- BD (5, 4)
- reform
- not reform
What’s the SPE in this game?

Someone show us on the board.